

# HKIA

## Study Guide for HKIA/ARB Professional Assessment Paper 3 Building Structures

The Hong Kong Institute of Architects  
香港建築師學會

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## Notes on the Use of this Study Guide

This study guide is prepared to assist candidates in their preparation for the HKIA/ARB Professional Assessment Examination in Paper 3 Building Structures. Topics covered in this guide are those that are likely to be addressed in the exam questions. The guide is not intended to be *comprehensive* in structural knowledge pertaining to building structures and their design, but is instead an *overview* of the topics and scope of structural issues that an architectural practitioner in Hong Kong should be aware of. Further study on the topics outlined below is encouraged and references are provided to direct the candidate to the most up-to-date and useful sources of knowledge about building structures.

The content of this study guide generally falls into two categories: general knowledge of structures (theory) and the practical knowledge of structure required for the safe design of buildings.

In this guide some of the information that is considered non-essential for the examination is printed on a gray background field, as indicated here.

Questions on the exam do not refer directly to this content.

Also particular case studies and examples of problem solving, design decision making or force calculations that might be considered as useful knowledge for a deeper appreciation of structure but not requisite for the examination are framed in boxes; green for case studies and orange for the latter.



To assist with candidates revision, questions are provided at the end of each subsection. And finally, reference to specific publications or other sections of the Study Guide are indicated as follows: *(Reference: Code of Practice Wind Effects in Hong Kong, HK Buildings Department 2019 Table 3-1)* .

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## The Role of the Architect in the Design of Structure

Before the establishment of the separate disciplines of engineering and architecture, there was only the architect. Knowledgeable about design and aesthetics, the architect was also trained in the practice of construction. Most often he was a master of the craft of building and designed the building to conform to current standards of “firmitas, utilitas, and venustas”, that is, strength, utility and beauty.

Today times have changed. We live in a world of rapidly developing technology and an expanding range of both choices and challenges. As a result, the professions are dominated with specialists who are focused on more and more specific areas of expertise within the building trades. This leads to many more professionals being involved in the design and construction of a building than ever before. Who then has the responsibility to lead and coordinate this growing team of experts?

Structure is one of many integral technologies required in building design. It is perhaps the most fundamental; without structure a building does not stand up. As the architect designs a building he/she is confronted by the demands of structural support. Just as a percentage of the overall cost of a building, foundations and structure are huge, and can be as much as 40% to 50%. The structural engineer is one of the most important consultants. The advice of an experienced engineer, especially at the outset of the project, can often save the architect the cost of the fee. Decisions about the viability or economy of certain choices related to structure are made in the early design stage. How then can we ignore the impact of structure on a design?

As architects we learn about structure not to replicate the role of the structural engineer but rather to enhance their ability to offer guidance as well as solution. Solution is never a problem. A good engineer can make anything work. But a gifted engineer will offer the architect alternatives that make better sense, both economically and often, aesthetically.

Our familiarity with the language of structural design is essential in having a meaningful conversation with the engineer. Without it we will often fail to grasp the nuances that are being presented in a discussion of structural alternatives. We may misinterpret what is being offered, resulting in frustration and a loss of confidence to make a sound decision. We may also feel uninformed and incapable of asking questions. The potential of a collaboration and exchange of ideas will not exist and the design process will reach an impasse.

What do we need to know about structure? Basically we need to have a science-based *understanding* of structural performance. As professional architects we should be able to differentiate between structures that have a chance of standing up versus those that have no chance! If we want to go further and participate in the conversation with the engineer, we should be familiar with the basic principles of structural design. What a load is. How loads are transferred in a structure as forces. What structural forms are appropriate for different performance driven objectives. And, most important, we should have a big-picture understanding of the building process from the foundation in the ground to the completion of enclosure.

The foundation is the starting point. We engage with specialist geotechnical engineers to discuss the alternative choices of foundation structure based on the conditions of the site and the nature of the building to be placed there. Will the building require deep piles that may necessitate either a wider column spacing or a transfer structure (or both)? Or can any structural framing be accommodated within certain limitations? Questions about the substructure may arise. What are the architectural implications? Soon we are discussing our concept with the structural engineer. As the architect, what do you propose? You are the initiator of the design and structure will be the main topic of your discussion. It will not be fully developed and will need a lot of input, but is it a realistic proposition? Finally as

the design evolves there will be complications. Architectural considerations may be confronted with structural requirements. Again the discussions with the structural engineer and other consultants will require collaboration and informed dialogue. In order to participate we must understand the language and be able to evaluate alternative solutions.

The study guide offers an outline of structure principles, an overview of different structural systems with brief explanations of how structural members react to forces and the implications this may have on design. Brief case studies are included to make a bridge between theory and application. Throughout this guide the goal is not to replace the expertise of a structural engineer but to provide a foundation for *informed collaboration*. After all, the knowledge and skills possessed by an engineer are gained through years of training and on-the-job experience that forms judgement. Our role as team leader must be to initiate and guide the discussion, and make the final decision. We rely on the engineering consultant to evaluate preliminary structural schemes and offer either confirmation or alternative suggestions. However, if we are conversant in the design of structure, there is a better chance that inappropriate and often costly decisions can be avoided and a design that achieves both visual quality and technical performance might be realized.

## Section A Structural Forms and Systems

### INTRODUCTION

*Structural Forms and Systems* introduces the range and basic characteristics of most of the common structure elements and systems used in building construction. This involves the broader issues of structural size and scale, classification of types and the definition of structural failure.

### TOPICS

#### 1.0 Types of Structures

In the building context, a structure is any device or mechanism for the transfer of loads through the building to the ground. We might refer to a single *element*, for example a beam, as a structure. A structural system denotes a group of elements working together, such as a floor framing system consisting of beams, girders, and decking, all inter-related and connected in order to transfer floor loadings to the vertical supporting structure, such as columns, piers or walls.

##### 1.1 What are some general classifications of structures?

- Linear versus surface, rigid versus non-rigid

The range of structural types may be distinguished from each other by their physical characteristics and form. We can identify certain structures as being *linear*, that is one dimensional, versus *surface* or two dimensional. Linear structural elements include beams and columns, arches, trusses and various cable supporting structures. Surface forms include walls, slabs, decking, vaults, domes, cable nets and membrane or fabric structures. Linear forms can be either straight or curved. Likewise, surface forms may be flat (planar) or curved.

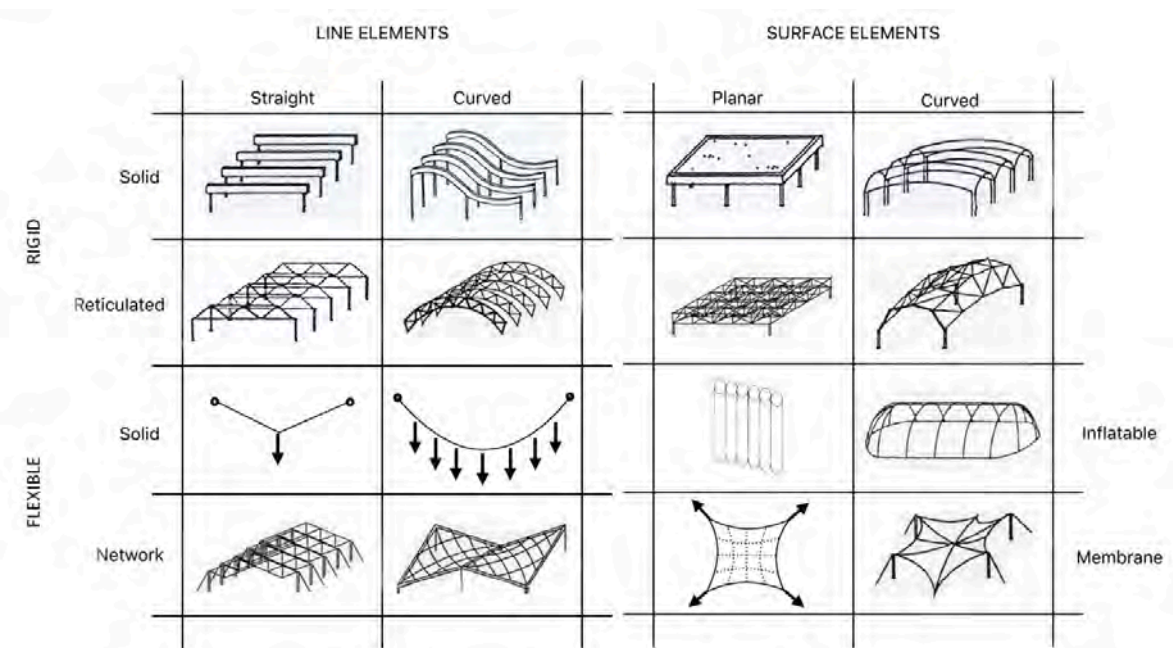


Figure A1.1: Form versus Rigidity

Another important physical characteristic of structures is *rigidity*. Most structures are rigid (stiff) and require considerable force to bend, stretch or deform. Cable and membrane structures are flexible and their shape is often determined by the action of loads. This property of *non-rigidity* allows these structures to also be categorized as *funicular*, based on their structural behavior.

- Solid, skeletal and surface structures

Individual structural elements can be assembled together in various ways resulting in very different forms of construction. Stacking of solid elements such as stone blocks results in a *mass* type of structure such as the ancient pyramids. Load transfer is essentially through *bearing*, the pressure of an element pushing down on another below it. This type of solid mass structure employs its own weight to provide stability. A gravity retaining wall is also an example.



Figure A1.2a: Pyramid of Khufu, Cairo 26th C BC.



Figure A1.2b: National Assembly Hall, Dacca  
Louis I Kahn. 1974

Beams and columns connected together form a skeletal or *frame* structure. The chief characteristic is linearity and thinness. As a result, skeletal structures under load must resist *bending* forces that will cause the structure to become unstable or even fail due to a lack of strength in the material. Skeletal structures can be composed of many small elements and configured in many forms, both orthogonal, geometric and free-form. A simple wooden post and beam structure is a skeletal structural as well as a large steel geodesic dome.



Figure A1.3a: traditional post and beam barn structure



Figure A1.3b Geodesic structure



Surface structures are a third type of constructed form. A surface structure is characterized by its thickness being very small in relation to its length and width. A flat plate slab is a simple two dimensional surface structure. Plate elements can be configured to form three dimensional structures that benefit from their non-planar geometry. An example is a folded plate roof structure. The category of surface structures also includes vaults, domes, and shells of various geometrical or free form shapes. A principal behavioral characteristic of rigid surface structures is their ability to transmit forces *in-plane*; their relative lack of thickness makes these structures especially vulnerable to concentrated forces applied perpendicular to their surface. Flexible surface structures such as membranes are limited to only tensile in-plane forces.



Figure A1.4a: A thin concrete shell.



Figure A1.4b: A cable-net roof membrane

- One-way versus two-way

Structures can be described as one-way or two-way based on the direction of the load-transfer mechanism. In a one-way structure the element or configuration of elements causes loads to be channeled in a single direction. A roof composed of parallel open web truss beams spanning between two walls is a one-way system. In a two-way system the direction of the load channeling is more complex involving multiple paths sharing different portions of the load. A square, flat plate slab supported on its four corners is a two-way system. Any load placed in the centre of the slab will be equally supported by each column, with the load paths radiating out in many directions.

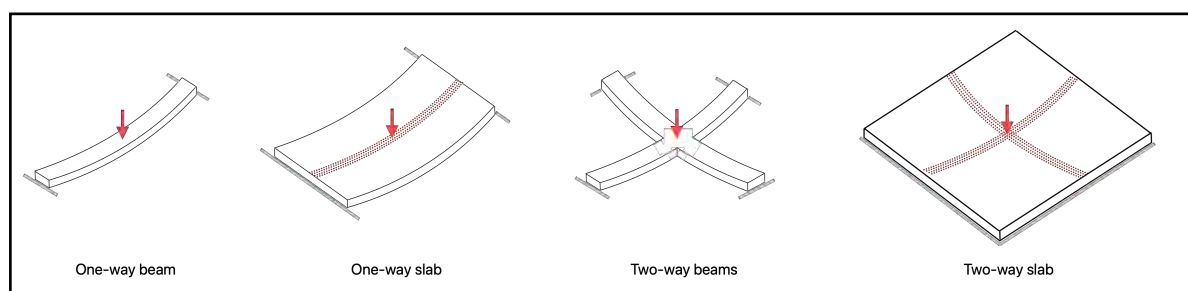


Figure A1.5: One-way versus two-way structure.

- Funicular versus bending

The internal structural behavior of a spanning element is dependent on several factors, including shape and geometry of the element and the manner in which it is supported at the ends. A beam resting on two supports at either end channels loads placed on it to its supports by a combination of bending stress (compression and tension) and shear. The primary force internally is bending which causes curvature of the beam. A flexible cable spanning between the same supports with the same load placed on it will channel or carry

the load to the supports by tension alone. The cable assumes a unique shape depending on the position of the load and the length of the cable that we refer to as the *funicular* form. This form can be replicated with a rigid structure and that structure will also be funicular; it will have no bending force present internally.

		Typical structures in bending		Typical funicular structures				
		Type of loading and type of support condition	One-way structures	Two-way structures	One-way structures		Two-way structures	
					Rigid	Flexible	Rigid	Flexible
Concentrated loads	Parallel supports							
	Continuous edge supports							
Uniformly distributed loads	Parallel supports							
	Continuous edge supports							

Figure A1.6: Bending versus Funicular. *Structures*, Schodek/Bechthold. Ch.1 Figure 1.16

- Material classification

Finally, structures can also be grouped according to the material they employ: steel, wood, concrete, etc. Material selection, however, does not necessarily change the way in which an element such as a beam carries load. A wood beam carries load primarily through bending in the same way as a steel or concrete beam. Material selection will impact other aspects of performance such as deformation and deflection.

## 1.2 What are the primary structural types?

In general, there are two categories of structures: spanning structures and vertical supporting structures. In addition there are various secondary structural elements that ensure the stability of primary structural elements. These include bracing members, plate stiffeners, cable tiebacks, spacing elements (blocking), and connectors.

- Span structures

Structures that support load between two points are referred to as span structures. As we saw in the classifications of structural forms, some elements have similar characteristics (one way, rigid, bending) but have different forms. A good example is that of a beam

versus a truss. Both are rigid, one-way spanning structures. Whereas beams are generally monolithic and have solid webs, trusses are composed of many linear elements connected in a triangulated configuration that produces an open web. These formal differences result in different mechanisms for resisting the bending moment caused by vertical loads applied perpendicular to the span. Whereas the beam develops a pattern of internal forces of varying magnitude distributed over the section from top to bottom, the truss concentrates the forces into axial tensile and compressive forces in the individual members between the joints. While using less material, the load carrying performance is enhanced: trusses are capable of longer spans with less deflection.

All span structures have a particular range of span in which they are most economical. In addition, the potential span is directly related to the depth of the structure. This is represented by the span to depth ratio ( $L/d$ ). These parameters are typically presented in span charts and are useful for the initial estimates of member size and span capability, as well as comparing the relative economy and efficiency of different spanning structures.

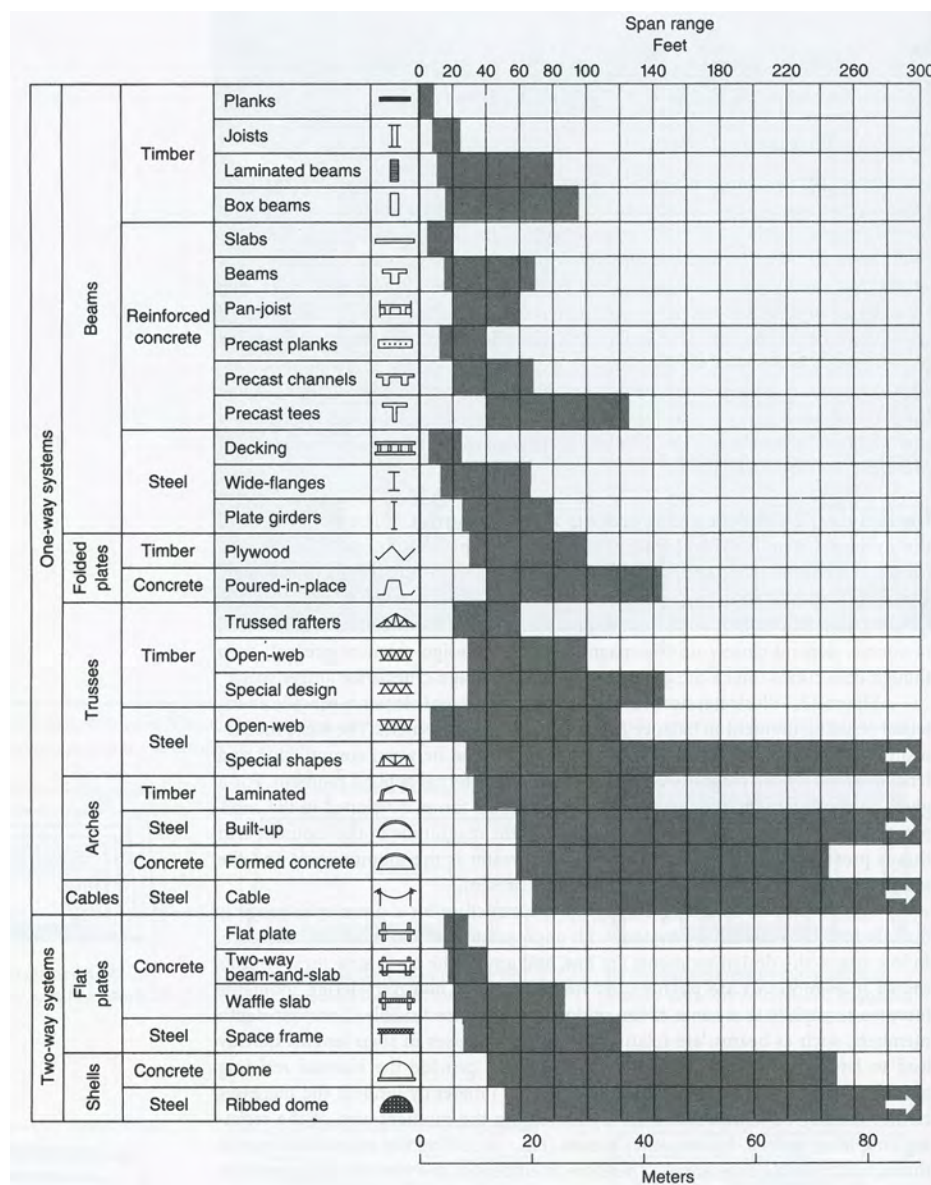


Figure A1.7: Approximate Span Ranges. *Structures*, Schodek/Bechthold. Figure 13.3 p426



- Vertical support structures

Most span structures are supported at their ends by a vertical support structure such as a column, wall or pier. Sometimes a span structure will be supported by another span structure, such as a beam or girder, or will be attached and bear directly on a foundation structure in the ground. Columns, walls and piers carry loads primarily in axial compression, although some bending forces may be present. These bending forces make the design of vertical support structures more challenging as considerations of instability caused by the potential *buckling* of the member may be critical and must be assessed.

## REVIEW QUESTIONS

- Review the different classification charts in the reference reading. Know the parameters of each structural element regarding its position in the chart.
- What are the basic economical span ranges of the most common structural elements?
- How does the ratio  $L / D$  help in the preliminary design of a span structure?
- Identify some non-rigid structural systems.
- What is a more efficient spanning structure? A beam or a truss? Why?
- What is the fundamental difference between a bending structure and a funicular structure?

## 2.0 Structural Behavior

Structural behavior refers to the response of a structure to applied loads, specifically, the development of internal forces and manner in which the structure will deform or deflect. As loading is increased on a structure the internal forces also increase, stressing the material of the structure to the point of failure or causing instability leading to failure. Predicting structural behavior is critical to the safe and economical design of a structure.

### 2.1 Structural Failure

Structural failure can occur in several ways. An error in the estimation of loads during the design process can sometimes occur but this is generally rare. More common is a situation in which a building structure is overloaded either during construction or after completion. Sometimes nature plays a role by imposing an unpredictable, once in a lifetime loading such as an earthquake or a very powerful typhoon. These unexpected loadings may cause structural failure in one of three ways.

- Strength

A strength failure occurs when the internal forces in a structural member create stresses that exceed the maximum strength of the structural material. For example, internal axial compression force in a wood column might create compressive stress that exceeds the compressive strength limit of the wood causing it to crush and split, leading to failure of the column. Strength failures can also occur in the *connections* of structural elements such as the shear failure of a bolted steel beam to column connection.

- Stability

The assumption that a building is at rest and does not move is associated with its stability. If a structure overturns or slides off its foundations we call this an overall stability failure. Sometimes a structural element, for example, a column, has a slender proportion that gives it a tendency to bend excessively and abruptly under a load. We say that it has buckled. This is a type of stability failure that occurs in a single member and can lead to the collapse of a building.

## Design Exercise: Selecting an economical span structure and determining its depth.

Given: Span is 10m. The structural material is reinforced concrete.

1 Observe which structures are economical at a span of 10m (the red line must pass between the diagonal shading bars that indicates the economical span range. Furthermore, the vertical line on the bar represents a typical span for the member. This is the most economic choice.

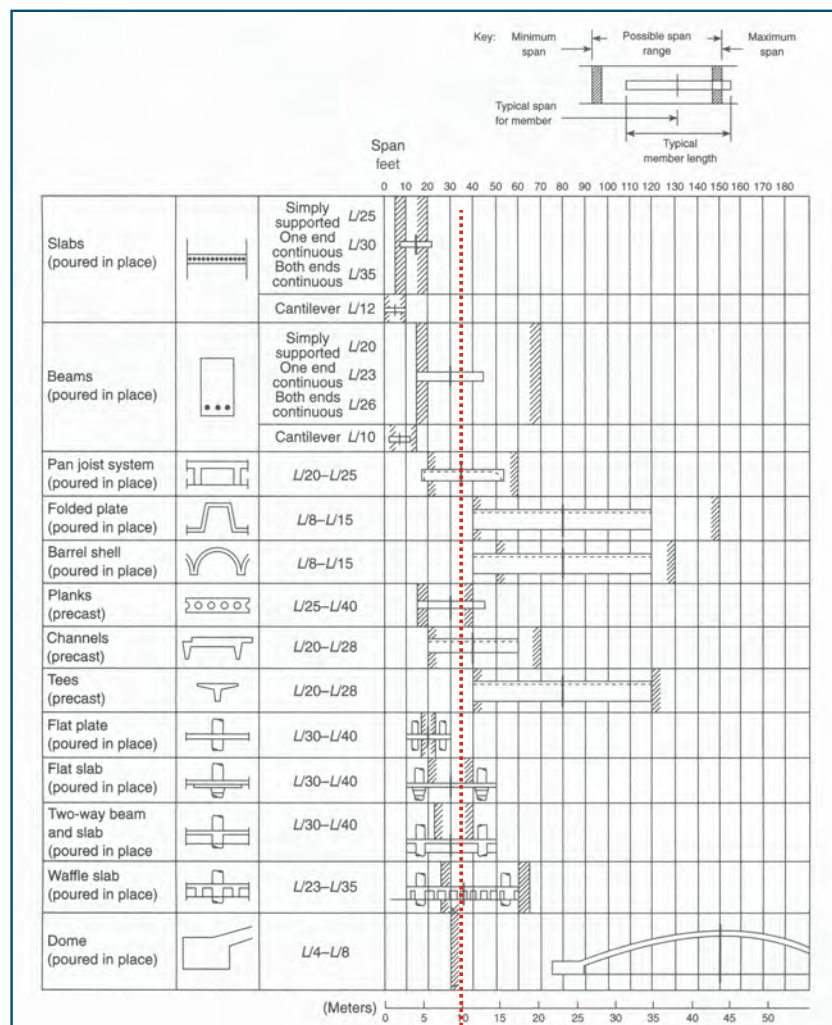
2 The possible choices are:

- A. Beam      B. Pan joist system      C. Precast planks      D. Precast channels  
E. Flat slab      F. Two-way beam slab      G. Waffle slab

3 The pan joist system (B) and the waffle slab system (G) both have typical member spans very close to 10m. This implies economy. The other choices are feasible but will tend to be less economical. The pan joist is a 1-way span system and the waffle slab is 2-way.

4. The span to depth ratio for the pan joist system has a range of  $L/d = 25$  to  $L/d = 20$ . The higher number (25) indicates the ratio that produces the smallest member depth. Since 10m is in the lower range of the span for the pan joist (approximately 6-17m) we can interpolate between 25 and 20 and use say 23 as a ratio. We then divide the span (10m) by the ratio (23) to determine the approximate depth (d).

$$10\text{m} / 23 = 0.435\text{m} = 435\text{mm}$$



- Stiffness

A structure firmly attached to the ground may move back and forth in the wind to such an extreme that inhabitants will find it unsettling and uncomfortable. The structure may not collapse but the excessive movement renders the building *unserviceable*, which is a type of failure. Individual elements in a building, for example a beam, can deflect too much because of a lack of stiffness. This type of failure may also not cause a building to collapse but the excessive deflection might cause damage to other non-structural building components. This would be considered a member failure due to a lack of stiffness.

A structural designer must anticipate all possible loadings that may occur on a building or any of its parts during its lifetime and predict how the structure will behave. In this way structural design measures can be introduced to prevent any type of failure.

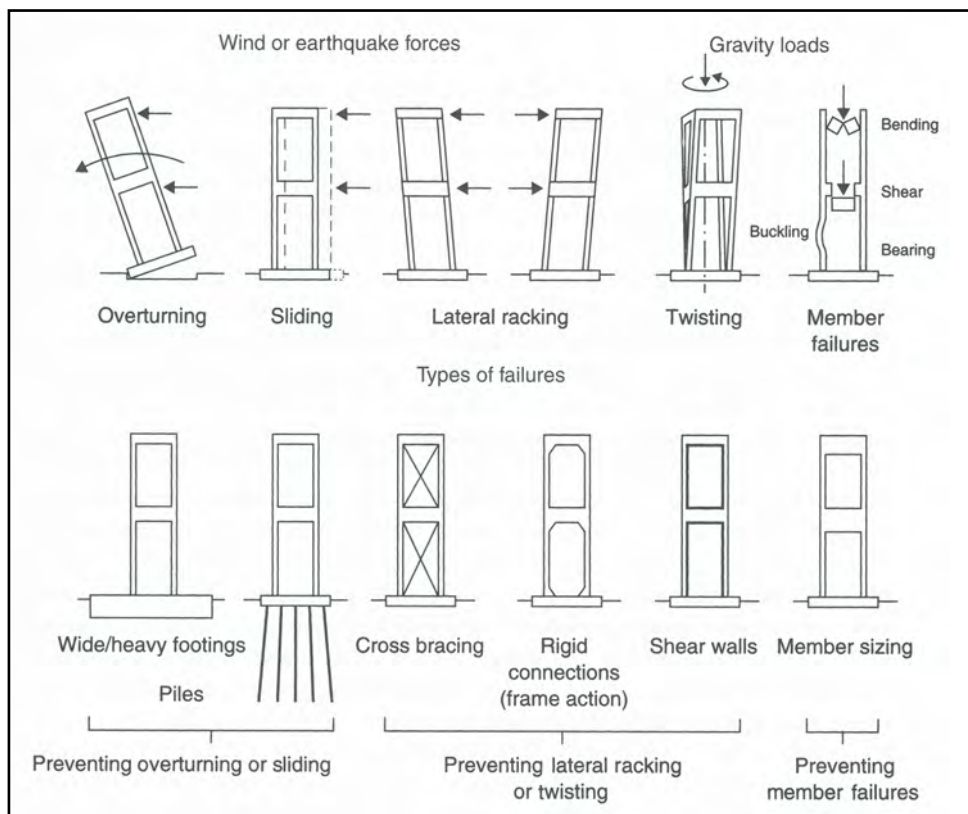


Figure A2.1: Types of Failures and Structural Responses for Prevention.  
Structures, Schodek/Bechthold. Ch.1 Figure 1.5

## 2.2 Structural assemblies

A structural unit is an assembly of several structural elements that can be repeated to form a building or portion of a building. An example is a structural bay framing unit. The bay might be two-dimensional and part of a floor framing system or it may take the form of a three-dimensional structural bay such as a unit bay formed by parallel portal frames.

- Post and Beam

The most basic of all structural assemblies: a simple beam supported by two columns or posts. The beam is not rigidly connected to the posts and therefore the unit lacks lateral stability. A post and beam assembly is more common in wood construction.

- Frame

A frame is also an assembly of beams and columns with connections that are either pinned or fixed. Frames with fixed connections are considered rigid and are able to resist lateral loads. Frames with pinned or hinged connections require additional bracing. Beams and columns in a frame system are subject to bending forces or moments. Frames are common to steel and reinforced concrete construction where the connections are rigid or semi-rigid.

- Trusses

A truss is a triangulated assembly of short, linear members connected with flexible hinge connections called “pins”. The geometry of the truss, that is its profile and the triangulated configuration of its members, is infinitely variable. The truss can be like a beam spanning between two supports. The connection of a truss at its supports is typically a hinge or roller type connection. Also like beams, trusses can form unit bays that are repeated to support large floor or roof areas.

Other structural assemblies can be “trussed”, that is, composed of many short pin-connected axial force members. Examples are space trusses, reticulated domes and vaults, trussed arches and trussed columns.

Because of the slenderness of axial force members and their pinned connections, trusses tend to be made from wood or steel, or a combination thereof.

- Arches

The traditional image of an arch is a curved spanning structure made of individual units of masonry (stone voussoirs). These units are shaped to fit tightly together without any connective device although sometimes mortar may be used to set the masonry. The shape of the arch in relation to the position and magnitude of its loads determines if the arch carries its loading through axial compression with no bending forces, or whether some bending force or *moment* is present. In a masonry arch, bending forces cause the individual masonry units to rotate and separate from each other, and sometimes lead to collapse. In a modern arch the material (steel, concrete, glu-lam wood, etc.) is continuous thereby giving the structure more rigidity and resistance to bending forces.

The connections at the ends of an arch vary. Fixed end connections increase the stiffness of the arch minimizing deflections. Hinged connections however are advantageous in allowing the arch to change shape very slightly (undetected to the eye) to prevent internal stresses developing in response to temperature expansion or uneven settlement of the support structure.

Arches like trusses can be repeated parallel to each other to form structural bays. Modern arches in steel or wood may be solid or trussed; the behavior does not significantly change.

- Plates

A wall is a type of plate structure in a vertical position. Its length and width dimensions are much greater than its depth or thickness. Therefore it resists loads acting in its plane and has less resistance to out-of-plane (perpendicular to the surface) forces. A flat slab is a horizontal plate structure. In this position it must carry loading perpendicular to its plane and therefore has internal bending forces and deflection.

Like an arch, a plate structure can be either solid or composed of many short, rigid line elements in a three dimensional triangulated pattern. This is sometimes referred to as a

space grid or space frame. The space frame is deeper than a solid plate structure but it is stiffer and can span much greater distances.

Individual plates may be connected along their long edges to produce a folded plate structure that can span long distances horizontally like a beam.

Horizontal plate structures can be made of steel, wood or concrete. Folded plates are generally made of concrete as it is easier to rigidly connect the plates along their edges. Walls can be made of any material able to resist in-plane compressive forces.

- Cylindrical Barrel Shells and Vaults

Cylindrical barrel shells and vaults are essentially singly curved-plate structures. A barrel shell spans longitudinally with curvature in the transverse direction. Vaults, on the other hand, span the short direction and are like a continuously extruded arch. Vaults carry loads in compression, but barrel shells behave like a beam with bending forces causing curvature and deflection.

- Shell Structures

Spherical shell structures are in the class of doubly curved surface structures that also includes warped surfaces (hyperbolic paraboloid and conoid shells). A spherical shell structure (a dome) may be a hemisphere or some portion of a sphere. It is often visualized as a rotated arch but the behavior of a spherical shell is very different from that of the arch. Spherical shells structures develop circumferential forces that do not occur in arches. Some shells like domes may be made of masonry units, but most contemporary shells are concrete. A shell can be trussed like an arch instead of being solid. It will have less weight but is capable of spanning very large distances.

- Cable Structures

Cable structures belong to the category of flexible tension structures. They are considered funicular structures by definition; they do not resist bending forces and carry load only by axial tension. There are basically two types of tensile cable structures: *suspension* and *stayed*. Cable stayed structures have a single length of cable connected between two points. One end is anchored or attached to a support while the other end carries a load. A cable stay is similar to a *tie-rod*. In a *suspension* structure the cable supports weight along its length and is deformed into a specific shape determined by the magnitude and placement of the loads. In either the cable stayed or suspended structure, the shape of the cable is referred to as the funicular form. If a cable supported between two points carries a continuous loading (e.g., the self-weight of the cable) the funicular form has the shape of a *catenary*.

Cables for stayed or suspension structures typically use high-strength steel wire strand cables or steel rods (for stayed structures).

- Membranes and Cable Nets

Membrane structures are made from thin, flexible sheets of fabric (e.g., PVC coated polyester or glass fiber) that are connected together to form continuous surfaces of varied shapes. These surfaces have double curvature in order to achieve stiffness and resist fluttering from wind forces. Double curved surfaces either have curvature in one direction like a sphere (synclastic), or in opposing directions like a saddle shape (anti-clastic). Because they are flexible, the shape is maintained either by a rigid structure to which it is attached, or by cables that pull the membrane (pretension) into its desired shape. Membranes can also be held in position and shape by air pressure and are called *pneumatic* structures.

Cable nets are structures of multiple suspension cables arranged in a grid configuration or mesh and connected at the crossing points or nodes. They create a doubly curved shape that can support panels to form a continuous covered surface.

### 2.3 Economy and Efficiency in Structural Design

Economy and efficiency are two related but fundamentally different concepts. The selection of a structural design invariably considers economy in terms of cost. The total cost of a structure must be evaluated from several different aspects. First there is the cost of material. For steel the cost of standard structural elements is directly related to the weight. Comparison between different materials (steel framing versus concrete frame) involves other factors related to construction (e.g., formwork for in situ concrete), transportation and deployment, and time or speed of construction.

Efficiency refers to the *performance* of a structure in carrying load. The relationship between structural form and efficiency is an interesting and important topic. In general, the amount of material (by weight) required to support or carry load is the main criteria for evaluating the efficiency of a structure. Structures that carry load with axial force (tension and compression) are more efficient than those which have bending force present. The funicular or *form-active* shape of a structure is one in which bending force is avoided. The concept of improved shape of a member in section and longitudinal profile is a second means towards increasing the efficiency of a structure. See the chart below that illustrates increasing degrees of efficiency in structural form.

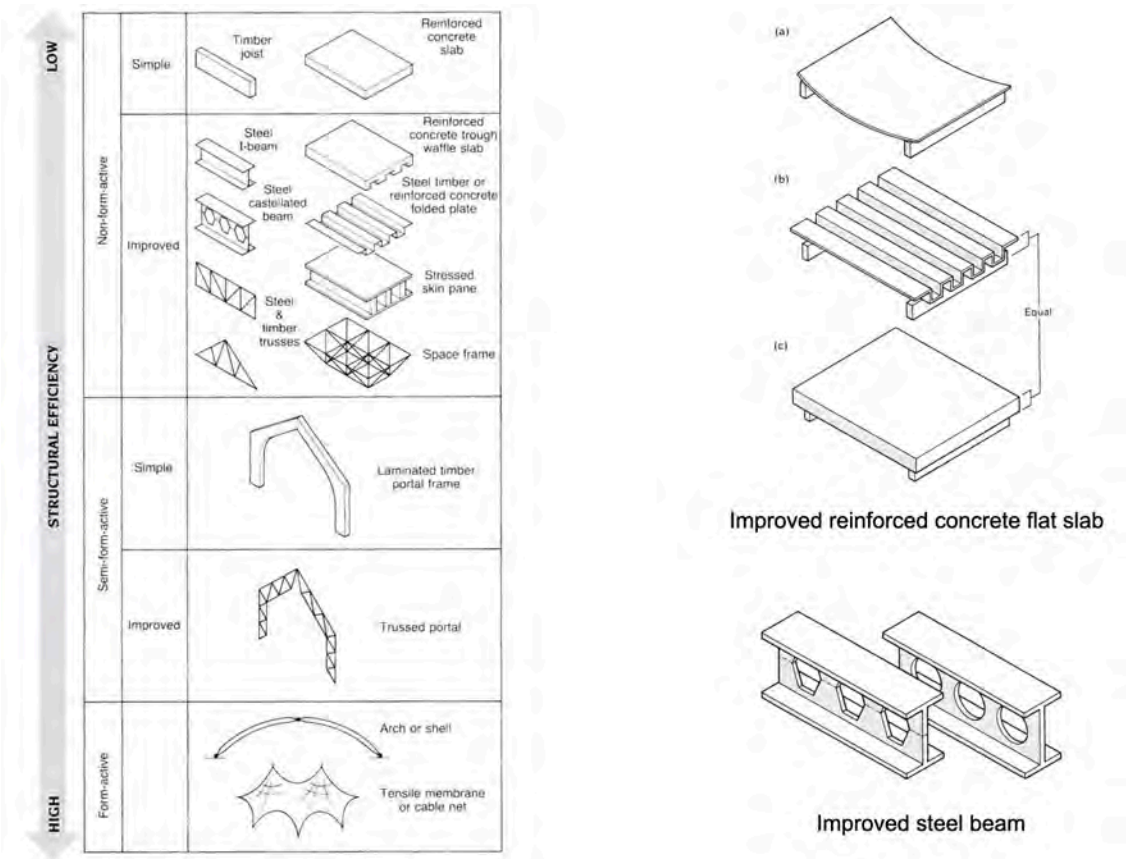


Figure A2.2: Structural Efficiency. *Structure and Architecture*, Macdonald. Ch.4 Table 4.1

## REVIEW QUESTIONS

- Identify the three conditions that can lead to structural failure. Give an example of each.
- Explain some of the differences between post and beam structure and frame structure.
- What is a “trussed” arch? Does it behave structurally like an arch or a truss?
- What type of connection is used to join the individual elements of a truss to each other?
- What is a funicular arch? What type of loading can be applied to a funicular arch?
- Describe a folded plate structure. Is it a span structure or a vertical support structure?
- What is the fundamental difference between a cylindrical barrel shell and a vault?
- Is a membrane dome structure synclastic or anti-clastic?
- Rank the following elements in terms of their efficiency as horizontal supporting structures. 1 is the least efficient. Parallel chord truss, semi-circular arch, circular closed section beam, universal beam, glu-lam beam, cable net with double curvature, gable frame, plywood decking, metal floor decking.

## SELECTED REFERENCE

- 1) *Structures*, Daniel L. Schodek and Martin Bechthold, 2014, Pearson. (Pt. I Ch. 1)
- 2) *Structure and Architecture*, Angus J. Macdonald, 2001 2nd Ed. (Ch.4]
- 3) *Structure in Architecture*, Mario Salvadori and Robert Heller, 1963, Prentice-Hall.

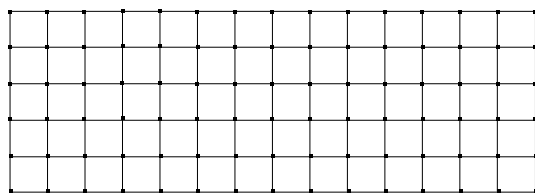


**Case Study: Wilkhahn Factory, Bad Münden, Germany. Thomas Herzog (Herzog + Partner).**

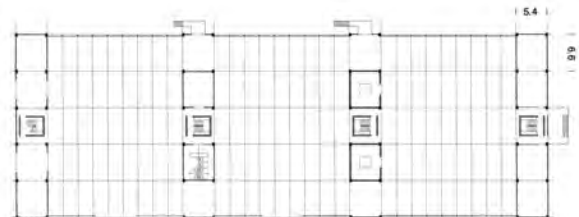
The furniture production factory for the company Wilkhahn that Thomas Herzog designed in 1992 can be used to illustrate the evolution of an efficient and expressive structure.

The program required open plan production space and smaller, enclosed offices and support areas. The site has a gentle slope. The design establishes a concrete podium that accommodates the shipping services and acts a sub-structure and base for a wood frame superstructure above.

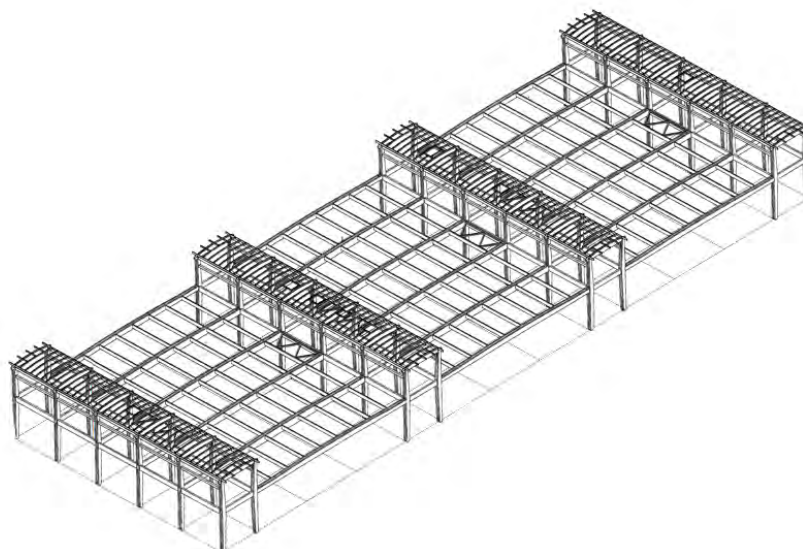
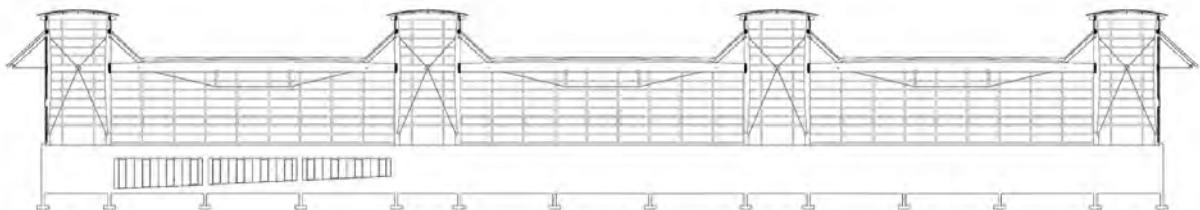
A starting point for the structural scheme might be consideration of a typical framing with equal repeating square unit bays. Recognising the spatial requirements of the program, an ABA variation in bay size is adopted early. The decision to use sustainable glulam for the framing allows for a long-span and deep beam for the large bay. Several strategies can be adopted to reduce the depth of this member and the amount of wood required. These are illustrated below in a series of diagrams representing successive improvements in both the efficiency and expression of the design.



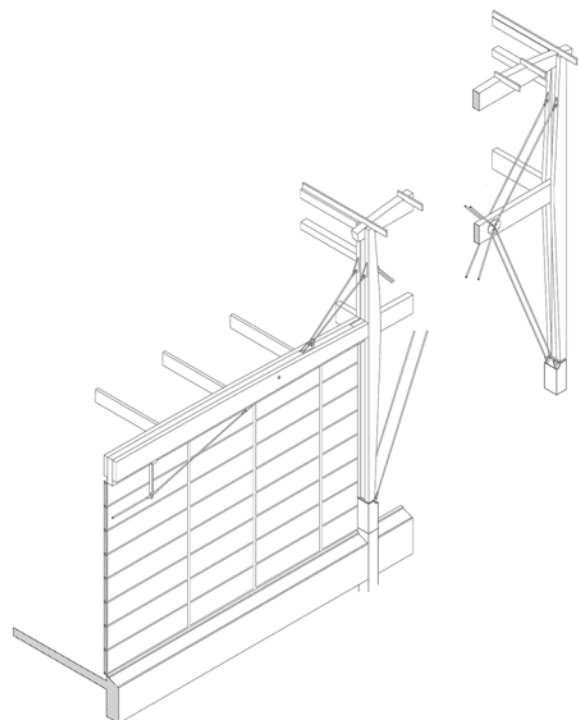
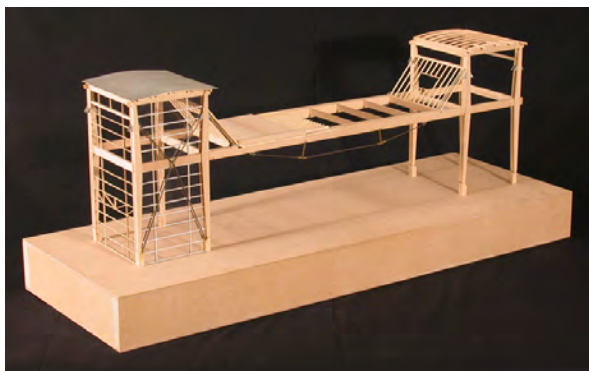
standard square bay column grid

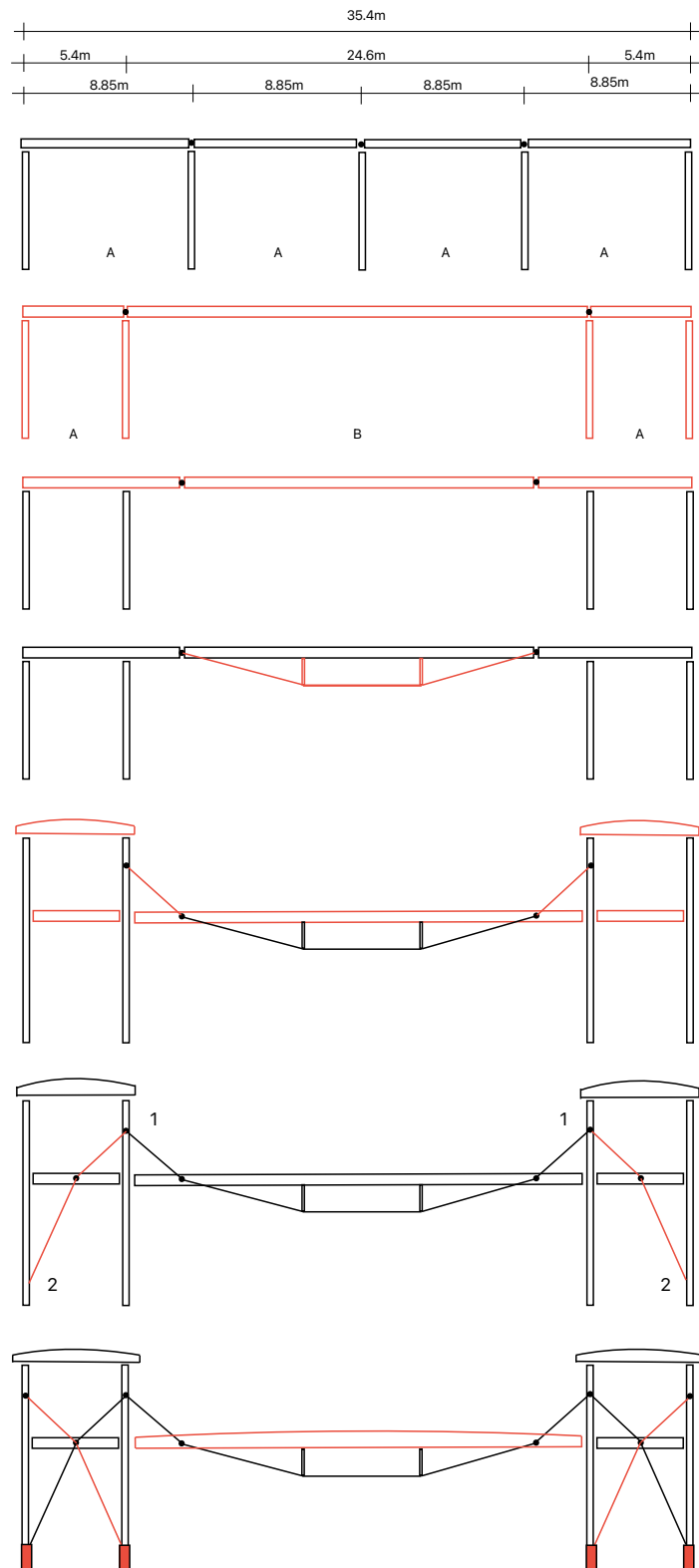


actual Wilkhahn structural grid with ABA bays









Step 1: Starting point. A square 3 bay wood frame. Post and beam wood framing requires some form of lateral resistance such as cross bracing or shear wall.

Step 2: Adjustment of span to an ABA bay frame to accommodate an open column-free production area and smaller bays for offices and services. However the middle bay exceeds the economical span distance even for higher strength glulam beams. Additionally, some type of lateral resistance still required.

Step 3: Introduction of a "Gerber" beam arrangement: two cantilevered spans with a simply supported beam between the ends of the cantilevers. This shortens the span of the beam, reduces bending moments overall and also provides some lateral stability to the frame.

Step 4: Improvement of the mid-span beam with steel tension cables and two compression struts to create a trussed beam, capable of greater strength and stiffness.

Step 5: Extension of the tension cables through the beam and upward with attachment to a raised second level of the smaller bays. This provides two points of vertical support at the ends of the trussed beam eliminating the need for the cantilevered beams. The continuous columns of the new tower elements in conjunction with the beam framing of the towers provides some stiffness to the overall frame, but insufficient to resist lateral force.

Step 6: Additional cables extending from the ends of "trussing cables" (1) resist the horizontal force these cables place on the column at this point, transferring the horizontal forces to the lowest portion of the column close to the ground.

Step 7: Adding cables symmetrically within the towers forms an "x-bracing" that provides the required stiffness and lateral resistance of the towers. The ends of the cables are anchored in concrete pedestals extending from the podium slab that also support the columns. The long span trussed glulam beam is given a slightly convex profile that increases its depth in the middle improving its bending resistance. The convexity of the top of the beam additionally facilitates a slight curvature to the roof that enhances water drainage.

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## Section B General Structural Principles

### INTRODUCTION

*General Structural Principles* presents basic concepts of how building structure resists and transmits natural and imposed loads to the ground. This study guide assumes a basic understanding of the physics of static forces. Certain key issues such as the transmission of forces in a structure, force equilibrium, stability and stiffness, mechanical properties of materials and stress, are explained and their relationship to structural design illustrated.

### TOPICS

#### 1.0 Building Loads

Building structures are designed to *support* the weight of all the people and objects that are on or inside a building, plus the self-weight of the structure. The building structure must also be designed to resist natural forces such as wind and earthquakes that impose loads intermittently on the building and its structure. All of these loadings must be *channeled* to the ground, mainly through foundations that are a part of the structural system.

#### 1.1 Types of loads on buildings:

<i>Vertical Loads (gravity)</i>	<i>Dead Loads: building mass (all permanent parts of a building including structure, permanent partitions, stairs, mechanical equipment and the building enclosure)</i>
	<i>Live Loads (also called imposed): people, movable partitions, non-built-in furniture, snow, water</i>
<i>Lateral Loads</i>	<i>Wind</i>
	<i>Earthquake (seismic)</i>
	<i>Sub-grade pressure (soil and/or water)</i>
	<i>Impact loads: blast, tsunami, avalanche or mud slide</i>
	<i>Other: Temperature (differential)</i>

#### 1.2 Static Loads versus Dynamic Loads

Loads that act on a building with the same force over time are considered *static* loads. They can be represented by a force vector or a force resultant vector (for loads that are distributed). *Dynamic* loads vary over time. An explosion imparts an instantaneous loading for a few seconds while wind is a pressure with variable intensity and direction over long periods of time. Some dynamic loads can be approximately modeled as a static force vector to simplify their calculation.

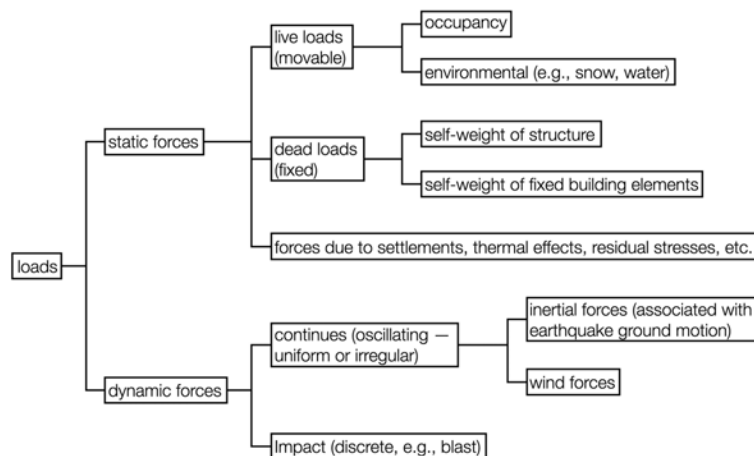


Figure B1.1: Load Diagram\_Static and Dynamic

### 1.3 Description of Loads

*Vertical loads* on building structure are generally the result of gravity. All of the building mass as well as any live (imposed) loads on the building is a downward force caused by gravity. Kilograms of mass become kilo-Newtons of force that are applied to the structure as either a *concentrated* load or a *distributed* load. Both are *static* loads and are represented as *force vectors* for structural analysis.

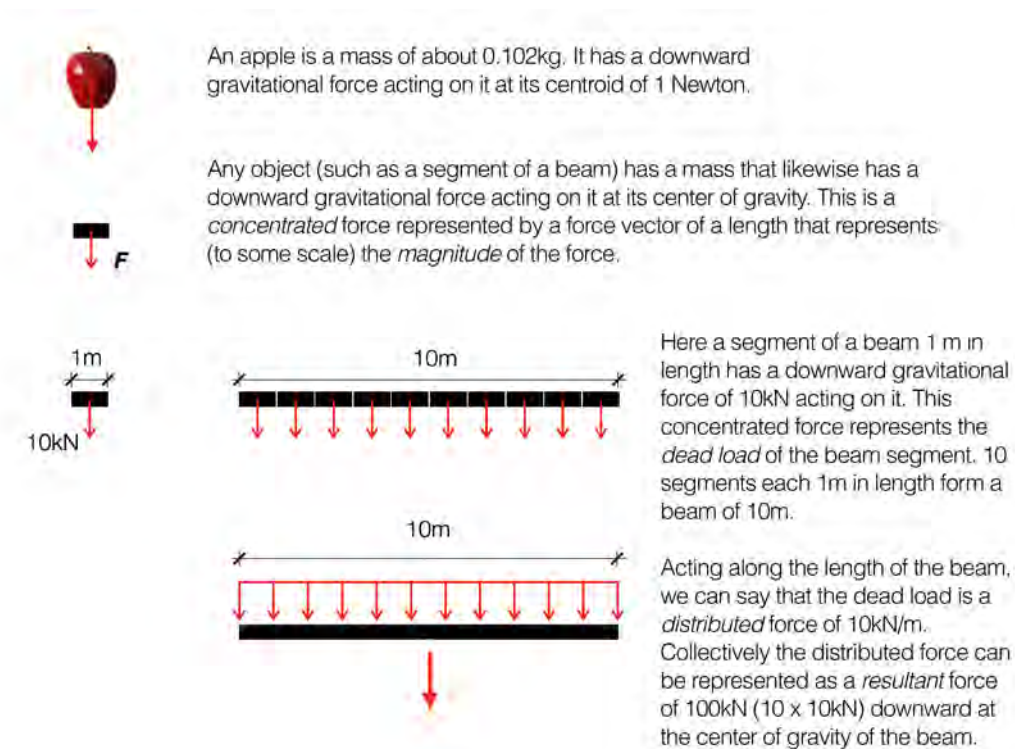


Figure B1.2: Loads as Force Vectors



*Lateral loads* produce horizontal forces on a building that can cause severe damage or even the collapse of the building. Wind and earthquakes are the two most important generators of lateral forces.

- Wind loads are dynamic in nature but are applied to buildings as a static wind pressure. Shown are typical wind-force coefficients for a gable roof building. Note the applied force is a percentage of the velocity pressure,  $q_h$  dependent on the position and angle of the building's surface. A negative pressure coefficient  $C_d$  indicates suction. Also note that the velocity pressure increases exponentially with the increase in wind velocity (air speed).

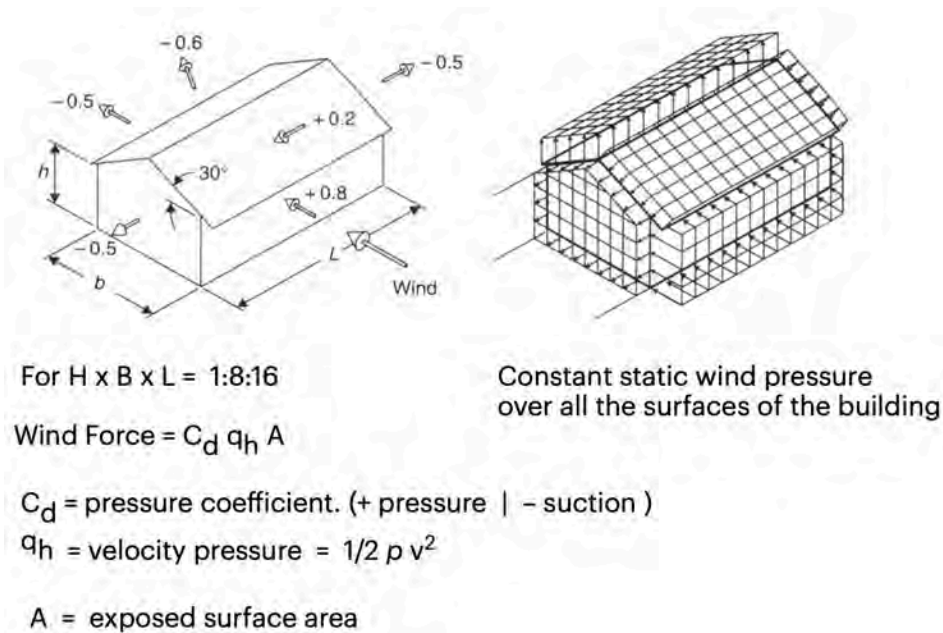


Figure B1.3: Wind-force coefficients. *Structures*, Schodek/Bechthold. Figure 3.4 p97

- “The wind load on any building, street, building works or street works shall be based on the response of that building, street, building works or street works to the velocity and gust effect of winds from any direction suitably determined from a return period of not less than 50 years.” (*Reference: CAP 123B Part III Regulation 18 Wind Loads. Hong Kong Building Construction Regulations*)
- The velocity of the wind is affected by two factors.
  - Context: urban building fabric, wooded country side, open landscape/water
  - Height: greater wind velocity at higher altitude

#### 1.4 Determination of loads

In order to design a structure to carry the loads of a building, the engineer must first determine what loads to consider and the magnitude and placement of the loads.

The *dead loads* (permanent) acting on the building are calculated based on an assumption of the weight of the proposed structure and all permanent non-structural elements (e.g., flooring material, mechanical services, window fenestration, etc.). (*Reference: Code of Practice for Dead and Imposed Loads, HK Buildings Department 2011. Appendix A Densities of Materials*)

Charts listing the weight per area and volume of construction materials and components are also useful for this purpose. (Reference: *Average Weight of Construction Components. Structures, Schodek/Bechthold. Table 3.2 p94*)

*Live loads or imposed loads* are determined by building code. *Occupancy loads* are the most common and are tabulated in the Code of Practice for various types of floor usage. There are eight classes of use. Two minimum imposed loads are provided. One is the minimum area distributed loading for the specified floor use ( $q_k$  in kPa). These range from 2.0 kPa for residential to 12.5 kPa for a printing plant. The second is a minimum imposed concentrated load ( $Q_k$  in kN) with placement anywhere in plan on a 50mm square area.

*Water and Snow live loads* are not considered in the Code of Practice for HK. Water and snow, if allowed to accumulate on a flat or slightly inclined roof surface, can create a dangerous structural loading. In Hong Kong snow is not present in any significant amount. Water is not allowed to accumulate on roofs or exposed deck areas and must be channeled off the roof by inclined roof surfaces, gutters and protected roof drains.

*Earthquake* resistant design is not required under the Hong Kong Building Code and earthquakes have rarely been felt in the Hong Kong region that are strong enough to cause major damage or loss of life. Nonetheless, it is important for Hong Kong Architects to be conversant in the basic principles of earthquake design since China is in a strong seismic zone. For a general review of earthquake design considerations, see: *Earthquake Design Considerations. Structures, Schodek/Bechthold. Ch.14.2 pp475-484*

*Wind loads* are a major consideration in the design of a tall building and therefore, in Hong Kong they are giving special attention. The determination of wind load uses an equation to calculate the wind pressure,  $Q_z$  :

$$(Q_z) = (Q_{o,z}) (S_t) (S_\theta)$$

velocity of the wind ( $Q_{o,z}$ )      topography factor ( $S_t$ )      directionality of the wind ( $S_\theta$ )

For tall buildings especially, the Code of Practice provides wind reference pressures for various heights. These are used in the calculation of wind force on the building. Note that the design wind pressure at the top of a 200m tall building is exactly twice the design wind pressure at grade level. (Reference: *Code of Practice Wind Effects in Hong Kong, HK Buildings Department 2019. Table 3-1*)

## REVIEW QUESTIONS

- Is a building lift considered a dead load or an imposed load? Rooftop solar panels?
- How is the dynamic loading of wind interpreted as a *static* applied load?
- What are two factors that affect the velocity of wind?
- Wind pressure on a building surface increases with the *square* of the wind velocity. True or False.
- The wind pressure on the surface at the top of a very tall building (400m+) is how many times the wind pressure at its base?

### Case Study: Collapse of a Multi-purpose Hall Roof Structure. City University of Hong Kong

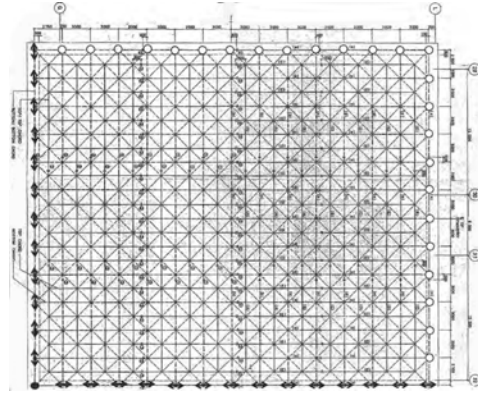
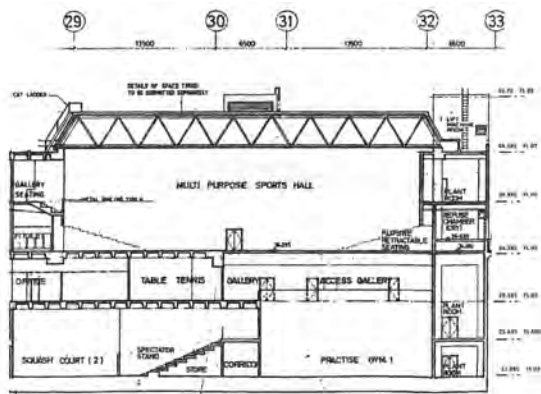
The roof of the Chan Tai Ho Multi-purpose Hall at City University collapsed on 20th May 2016. Three persons were injured. Cause of collapse: additional loading in the form of a new "green roof" which led to "ponding" of water during a heavy rainfall.. The steel space frame structure (42m x 36m x 2.25m) was originally designed to support a max. dead load of 4.75kPa and a live load of 0.75kPa. The additional loading of soil has been estimated as more than 1.30kPa, exceeding the design loading. The wetness of the soil after rain was a contributing problem.

"The Buildings Department (BD) today (31st May 2016) released the final investigation report on the collapse of the roof structure at the Chan Tai Ho Multi-purpose Hall of the Hu Fa Kuang Sports Centre at the City University of Hong Kong.

The investigation concluded that the collapse of the roof structure was caused by overloading, as a result of the following three factors: The screeding of the roof structure was thicker than the original design; The laying of greenery cover on the roof; and localised water ponding on the greenery cover.

The findings of the investigation revealed that the main structural element of the roof structure of the sports hall was a space truss system (STS). The STS provided support for a reinforced concrete slab that served as the decking of the roof. Screeding, an insulation layer, waterproofing membrane and the greenery cover were on top of the roof slab. The screeding, which was thicker than the original design, together with the laying of the greenery cover increased the loading imposed on the STS. As such, the gradient of the roof was reduced, which affected the rate and flow of water discharge, resulting in localised ponding on the greenery cover. Following the increase in the extent of water ponding on the greenery cover, the loading on the STS was further increased and finally exceeded the loading capacity of the STS, leading to the collapse incident."

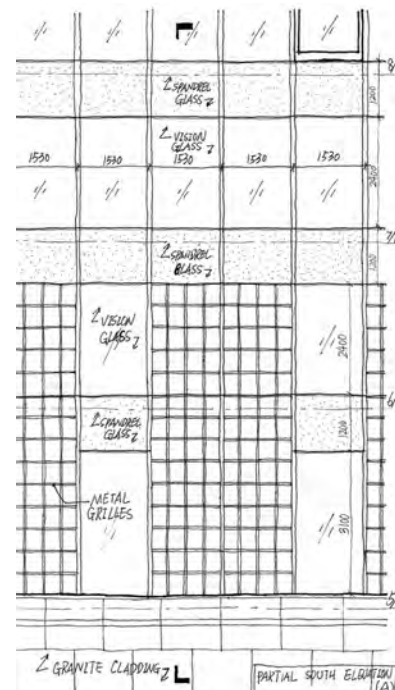
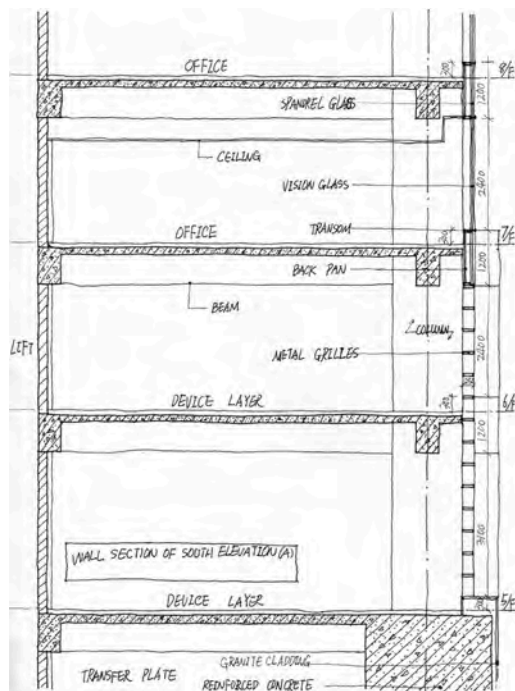
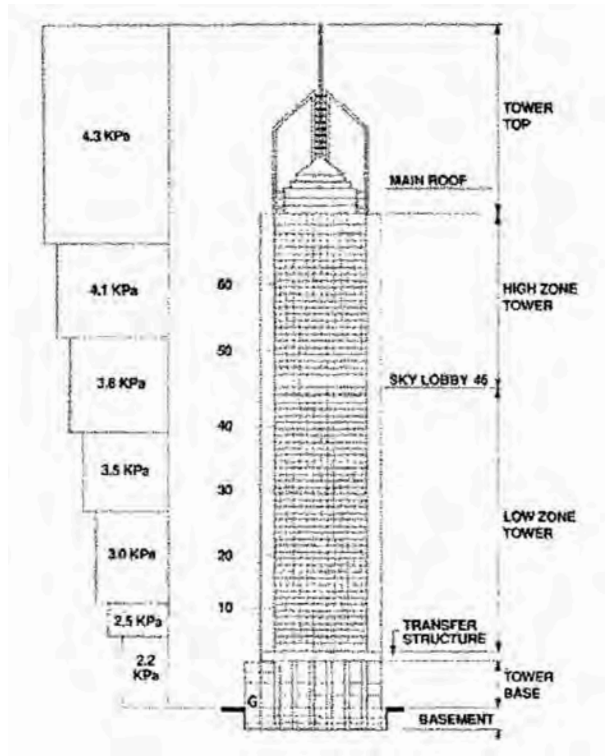
<https://www.bd.gov.hk/en/whats-new/press-releases/2017/0531A-buildings-department-releases-final-investigation-report-on-roof-structure-collapse-at-cityu-sports-hall.html>





### Case Study: Consideration of Wind Force in the design of the Central Plaza Tower

The Central Plaza Tower was the tallest reinforced concrete tower in the world when built. (78 floors at 374m). The cost of providing resistance to lateral wind force in terms of additional structure was estimated as approximately HK\$1 million for every additional 1 meter in height. Designers developed a very efficient floor to floor dimension of 3.6m to save money. Design wind pressures determined by the Code of Practice on Wind Effects 1983, Building Authority Hong Kong.



partial section and elevation



## 2.0 Load Path

Structures are designed to carry or *channel* all applied loads to the ground. The process of loads becoming internal member forces which are then transmitted through the building structure to the foundations and finally, into the ground, is referred to as a *load path*. The load path is not a linear, unitary path analogous to plumbing, but a complex distribution of shared resistance to the action of the loads that involves member stiffness, connectivity, and configuration.

*“Force follows the path of greatest resistance.”*

## 2.1 Structural Connections

Structural elements such as beams, columns, trusses, etc. are connected to each other and sometimes to the foundation or ground in various ways. Connections are a critical component of structural design. They affect member behavior and if improperly designed and built, can lead to structural failure.

Connections vary in terms of the amount of ‘fixity’ that they have on the ends of the structural member. A wood timber beam resting on a wall with no mechanical fastening is the most basic type of connection. The wall resists the downward force of the beam preventing motion in this direction. In every other direction the beam can be moved with a force applied. Also the end of the beam can *rotate* on the wall if support on the other end of the beam is removed. So we say that the end of the beam has one degree of restraint: downward. This would be an example of a simple support.

There are 5 types of connections or support conditions with various degrees of restraint.

Type of connection	Typical symbols	Types of translations and rotations the connection allows	Type of forces that can be developed at the connection	Types of forces that can be developed when the support is inclined
Fixed support				
Pinned support				
Roller support				
Simple support				
Cable support				

Figure B2.1: Connections and types of restraint. *Structures*, Schodek/Bechthold. Figure 2.15 p41

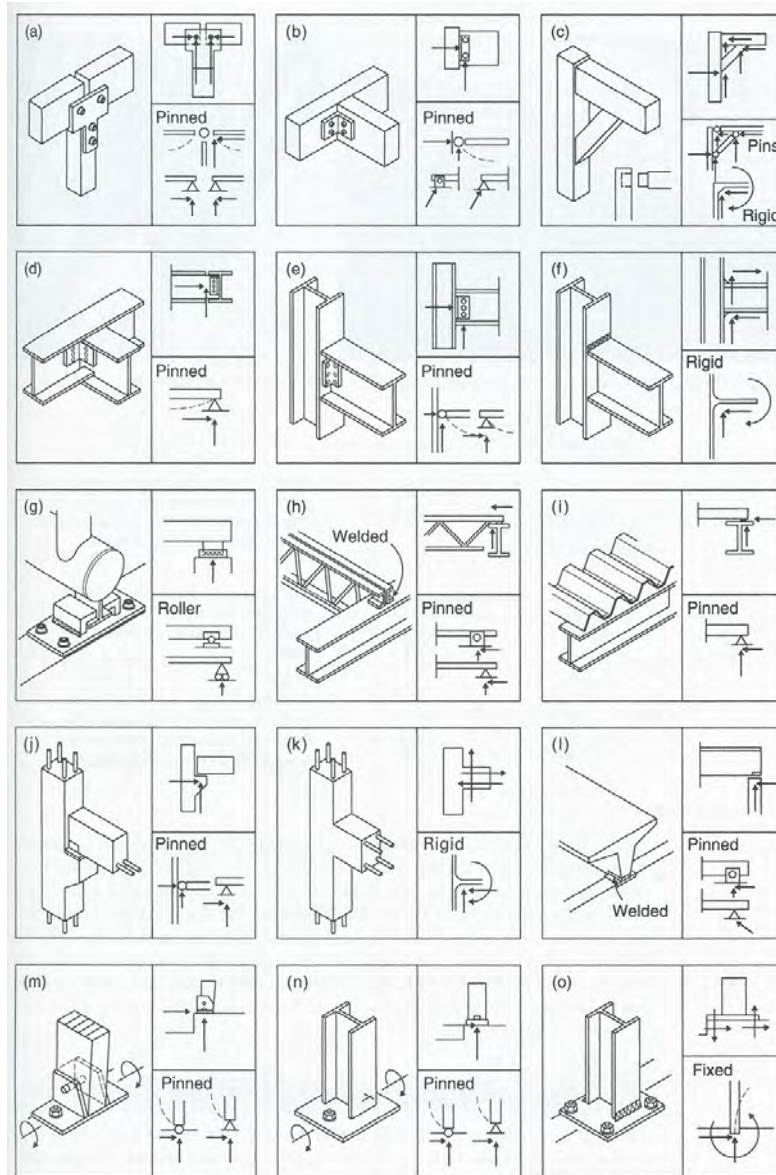


Figure B2.2: Types of Connections. *Structures*, Schodek/Bechthold. Figure 3.10 p105

## 2.2 Modelling a Structure

To study and analyze loads and forces on a structure, engineers use diagrams with abstract notation to represent the real situation.

- i. Linear members such as beams and columns are represented as a single line.
- ii. Applied loads are represented as force vectors. A distributed loading is represented by line on top of and connecting a series of small vertical lines.
- iii. Connections of structural members are represented as following:
  - fixed rigid connections between members: lines are simply joined together
  - hinged or pin connections between members: a small circle between the adjoining members
  - fixed-hinge reaction support: a small triangle on a short line with diagonal hatching underneath

- roller-hinge reaction support: a small triangle on a short line with two small circles underneath
- fully-fixed reaction support: a short line perpendicular or parallel to and attached to the member with diagonal hatching underneath

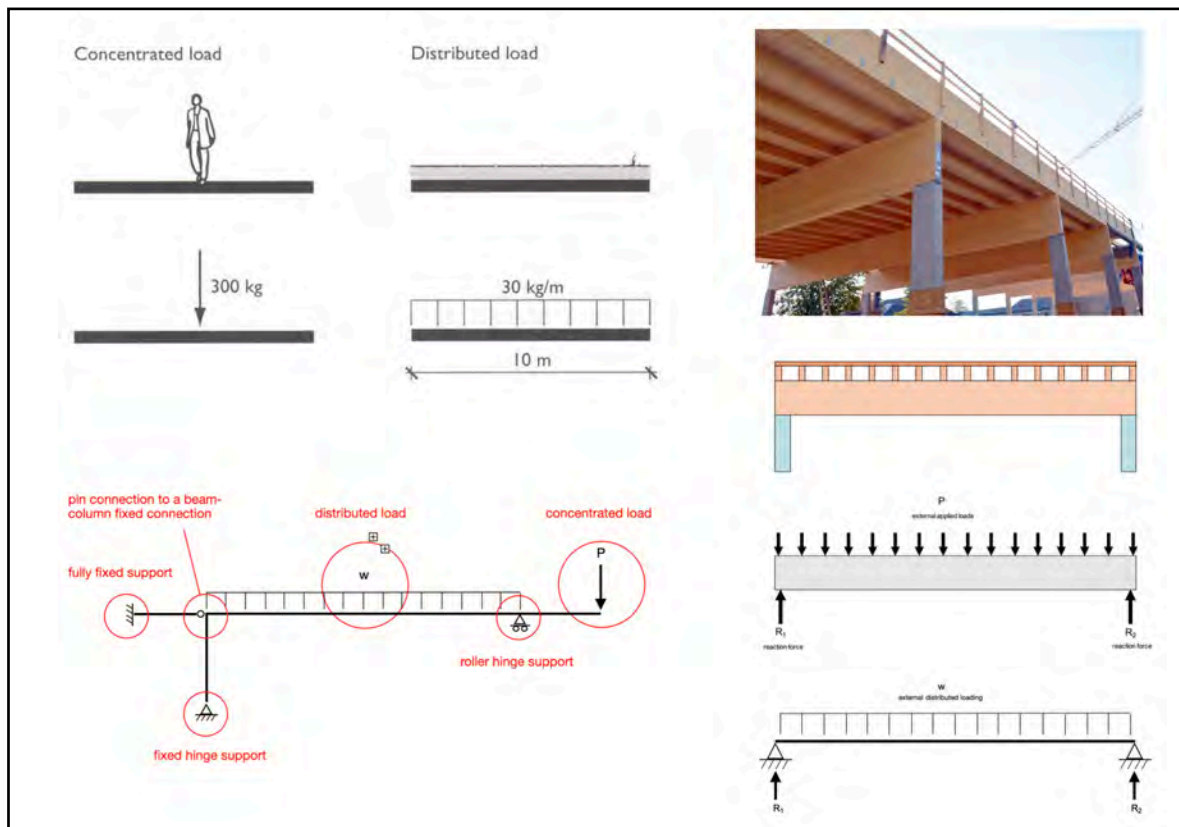


Figure B2.3: Modeling the Structure

### 2.3 Span Hierarchy

Many horizontal spanning systems, particularly those in steel and timber, are made up of hierarchical assemblies of different kinds of members. The hierarchy of members is determined by the load path. If a structure such as a concrete slab spans between two walls that provide vertical support, we call this a *one level* span system. In most cases, however, the horizontal span is composed of more than one structural spanning element, such as slab decking supported on beams that are then supported by a larger beam girders. This would be a *three level* hierarchy. Note that the hierarchical level does not refer to the positions of span elements in space. The load path may be above, below or even in the same plane.

Two-way spanning systems will tend to have span elements in the same plane (e.g. a r/c waffle slab). Examples include two-way flat plate slabs, various surface structures such as shells or membrane suspension structures, cable nets and pneumatic structures.

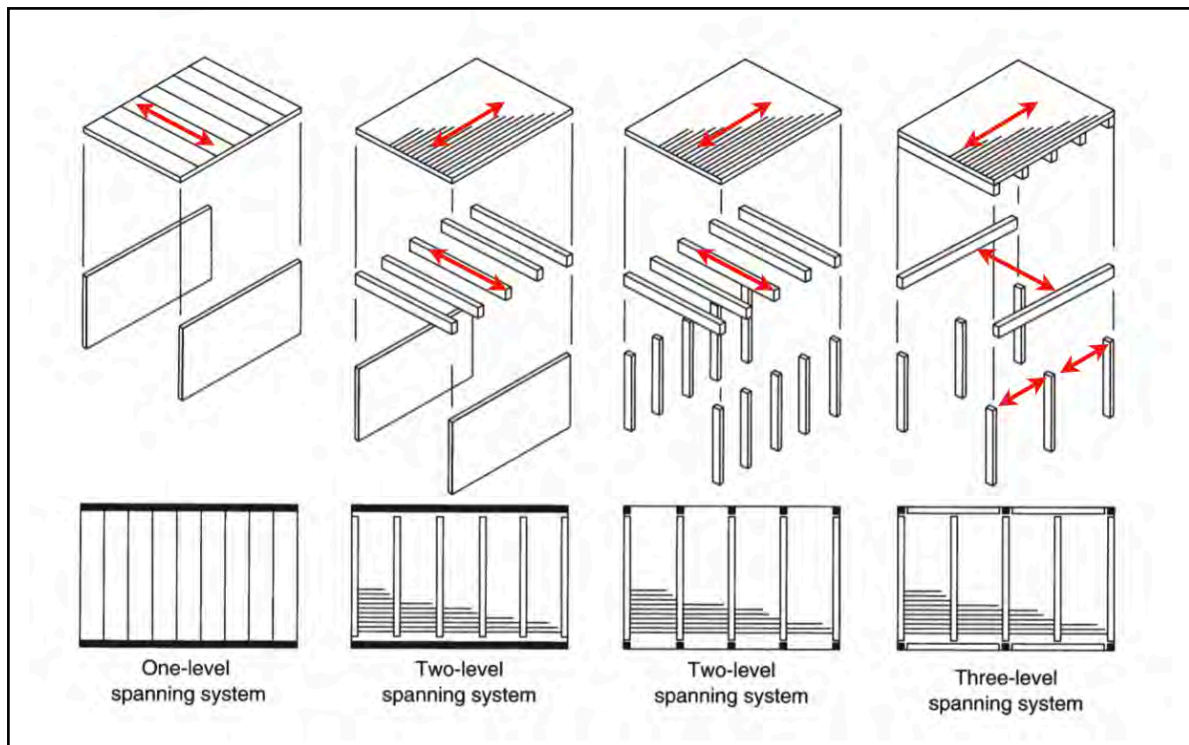


Figure B2.4: Span Level Hierarchy in One-way Span Systems. *Structures*, Schodek/Bechthold. Figure 13.7 p431

## 2.4 Method of Calculating Loads and Reaction Forces

There are two methods for calculating the loads on a vertical support member of a floor or roof spanning system. The first method is based on the *structural assembly*. First the live load is placed on a surface span element such as a beam plank. Half of the total live load and half of the dead load of the beam plank will be resisted by the supporting element it rests on. This force is called the *end reaction*. The end reactions of a series of planks supported by a beam constitute a series of concentrated loads along the length of the beam. Half of these loads in turn become the end reactions of the beam. And so forth.

A second method can be used on floor or roof spanning systems that are evenly repetitive and continuous. This method referred to as the *contributory area* method determines the total combined dead and live load of the whole surface area and then calculates the amount of load carried by the vertical support elements based on an equal distribution of the total load over the surface. Each vertical support carries an area of the floor or roof that is half the distance to the next support in each direction. (Reference: *Load Modeling and Reactions. Structures*, Schodek/Bechthold. Figure 3.13 p107)

## REVIEW QUESTIONS

- Determine the load on a corner column in a 3 x 3 bay flat plate framing plan with 6m column spacing in each direction and a total combined dead and live load of  $6\text{kn/m}^2$ .
- Will the loads on all the columns in a rectangular floor framing be equal if all the loads are equally distributed and the column to column spacing is equal?
- Describe the load path of a concentrated load on the roof of the Kemper Memorial Arena. Identify each span and vertical support element in the path of a load to the ground.

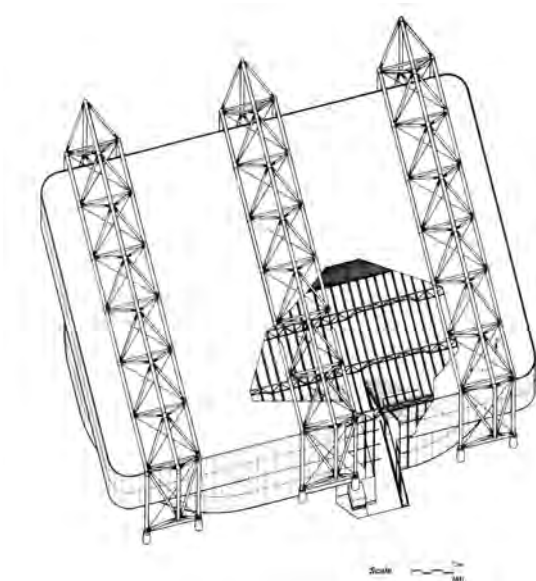
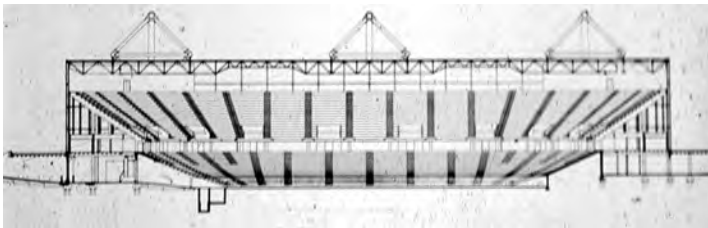
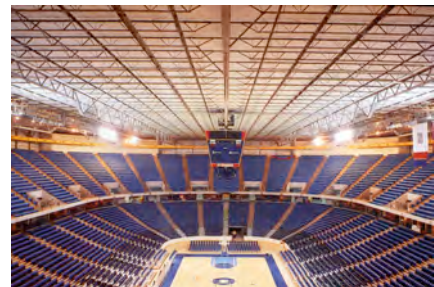


### Case Study: The Kemper Memorial Arena.

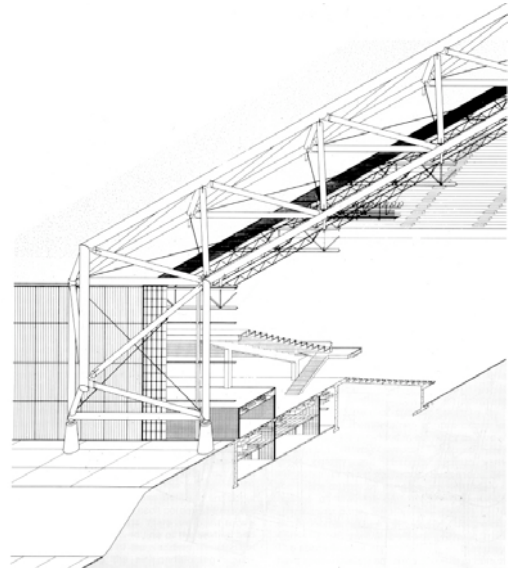
The Kemper Memorial Arena is a large sports hall designed by the architecture firm of Murphy/Jahn and built in Kansas City in the USA. It was completed in 1973 but suffered a roof collapse in 1979! Cause of failure: a fatigued hangar bolt and overloading caused by water ponding due to heavy rain.

Online reference: <https://kansascitymag.com/news/only-in-kc/40-years-ago-the-collapse-of-one-of-kansas-citys-most-iconic-buildings-shocked-the-architecture-world/>

Description: Steel 3D portal trusses (span 97m) with long-span steel trusses attached below the truss nodes. These support open web joists in the direction parallel to the main truss. The open web joists support steel deck. The dead load of the roof assembly is  $1.3\text{kN/m}^2$ . The roof is designed for a live load of  $1.25\text{kN/m}^2$ . The live load includes rain water, mechanical systems and other hanging loads attached to the roof. At each hangar point attachment to the main trusses the concentrated load  $P$  is  $622\text{kN}$ . The structural system has a four level spam hierarchy.



Cut-away axonometric



Cut-away elevation oblique

### Design Exercise: Calculate the load on a column using the *Contributory Area Method*

The wood frame structure shown has columns supporting trusses that collect beams that support a wood roof deck. This is a one-way system with even column spacing and no irregularities. In such a condition the contributory area method will provide the column loads directly.

1. Determine the uniform distributed loading on the structure. Simply add up the dead loads of the structural members and the live load.

- For the live load  
From Table 3.8 Class 7A Code of Practice for Dead and Imposed Loads:  
(Flat Roof inaccessible).  $q_k = 2 \text{ kPa}$  or  $2 \text{ kN/m}^2$
- For the dead load  
To be accurate, the dead load must be determined by a calculation of the weights of the individual framing members and the roof decking. In a design problem, the exact sizes would be unknown. Therefore the designer uses an approximate estimate from available charts for the dead load based on the type of construction and span.

For a typical wood truss and beam system with a span of 11m the dead load is approximately  $1.0 \text{ kN/m}^2$

Therefore the total distributed load of the roof structure is:

$$2 \text{ kN/m}^2 + 1 \text{ kN/m}^2 = 3 \text{ kN/m}^2$$

2. The plan of the roof framing shown below indicates the contributory areas of loading on the trusses. Each truss is supported by a column at either end, therefore a typical column load will be the weight transmitted by the truss or half of the contributory area. Using the contributory area we calculate the total weight.

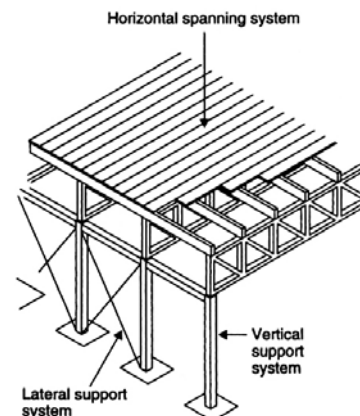
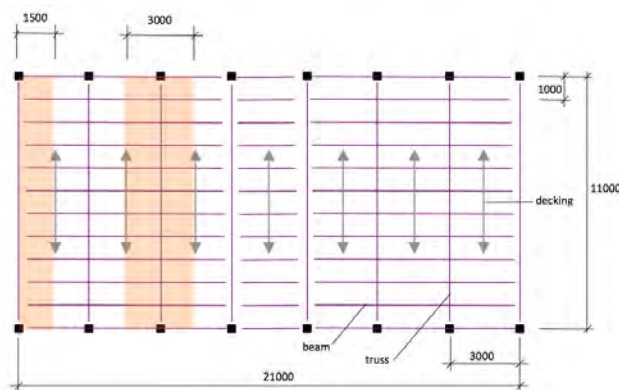
$$\frac{1}{2} \text{ contributory area} \times W_{LL + DL} = \text{column load}$$

for a typical side column:

$$\frac{1}{2} \times 3\text{m} \times 11\text{m} \times 3 \text{ kN/m}^2 = \mathbf{49.5 \text{ kN}}$$

and for a corner column:

$$\frac{1}{2} \times 1.5\text{m} \times 11\text{m} \times 3 \text{ kN/m}^2 = \mathbf{24.75 \text{ kN}}$$



### 3.0 Force and Stress

Externally applied forces on a structure are resisted through the development of internal resisting forces in the individual members of the building structure. Structural members *deform* as the result of stresses that are created by the internal resisting forces. These deformations may cause failure of the structure through the failure of individual structural elements or the instability of the overall structure.

#### 3.1 Statics

The term “statics” refers to a part of mechanics (a branch of applied science that deals with forces and objects in motion) that describes the interaction of forces and rigid bodies in *equilibrium* and at rest.

Loads on structures are represented as externally applied force vectors, both concentrated and distributed. By their position and action on the structure, these external forces create *internal forces* in the structural member that include Axial Tension and Compression Forces, Shear, Bending Moment and Torsion. In turn, these internal forces act on the material of the structure generating associated stresses that result in the *deformation* of the structure. Although these deformations are generally quite small (and hardly visible) they can initiate various types of structural failure that lead to permanent damage and sometimes collapse. For example, beam deflection is a member deformation caused by bending stress.

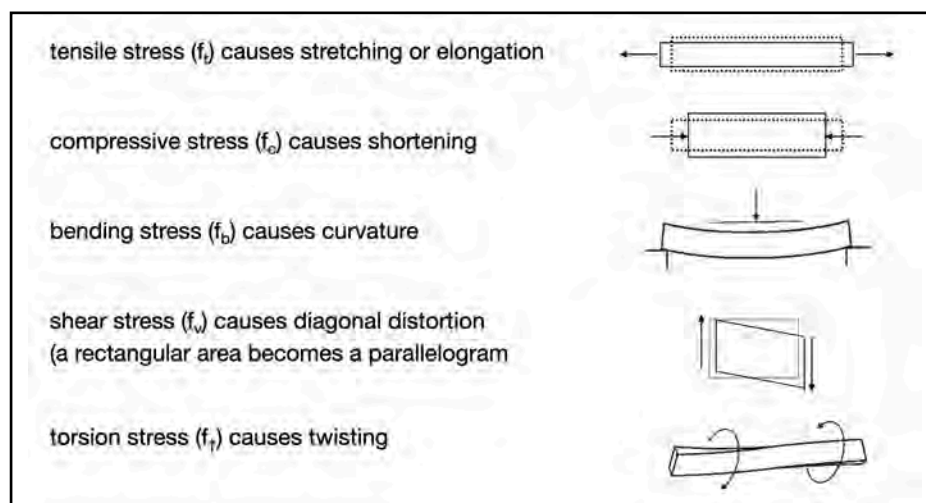


Figure B3.1: Types of Stress and Deformation

- Axial Tension and Compression

A flexible steel wire-strand cable is an example of a pure tension structure. It resists applied tension forces at its ends and generates internal resisting tension forces. These internal tension forces act across the section of the cable creating evenly distributed tensile stress. These tensile stresses will cause the steel cable to stretch or *elongate* slightly. Because the cable is flexible, it has no resistance to compression force or bending moment force. Therefore, only axial tensile stress is present.

Columns resist applied axial compression force which generates compressive stress over the cross section of the column. These compressive stresses will cause a concrete column to contract or *shorten* slightly. The amount of elongation of the steel

cable or contraction of a concrete column depends on a material property of steel and concrete called the *modulus of elasticity*.

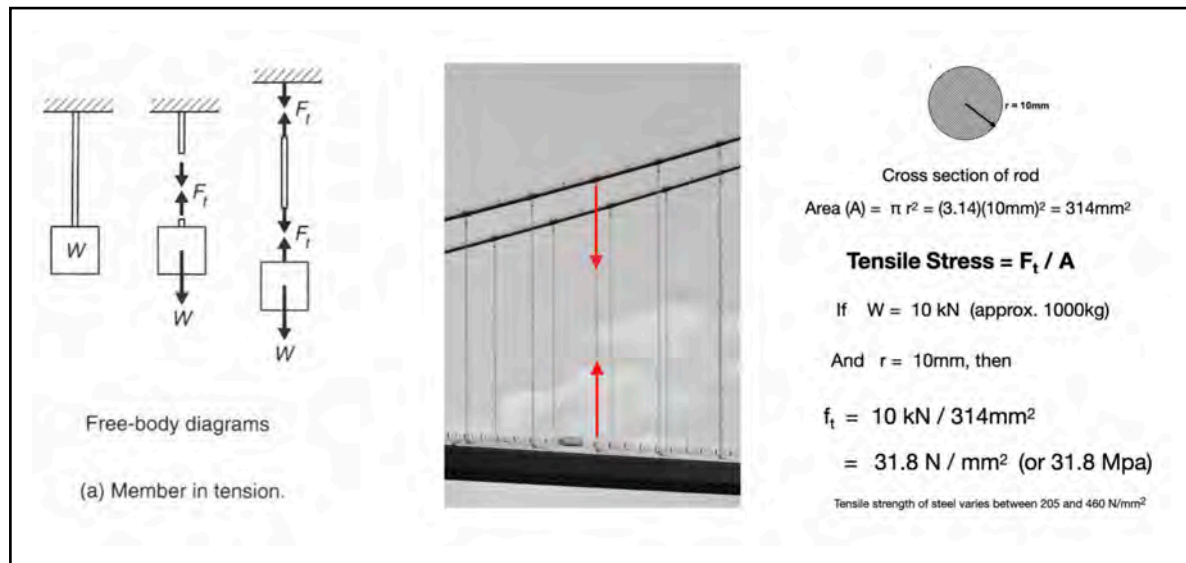


Figure B3.2a: Tensile Stress

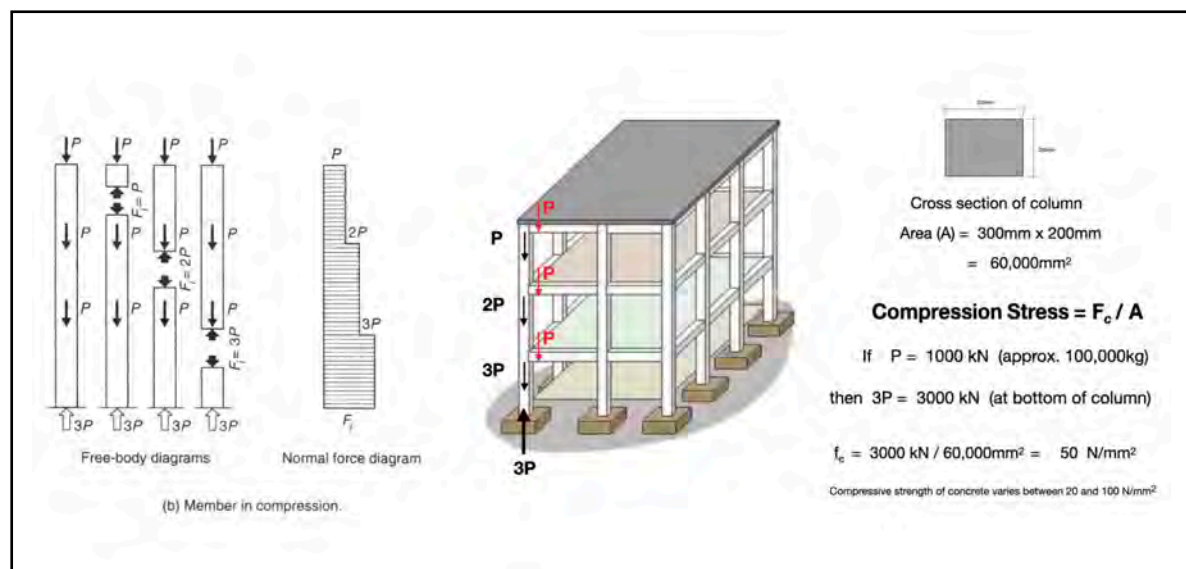


Figure B3.2b: Compressive Stress

- Shear

Internal shear force is created by external forces acting perpendicular or transverse to the axis of a structure. They attempt to move portions of the structure up or down on either side of a vertical shear plane. The stress generated on the vertical shear plane is *not* evenly distributed over the cross section, varying from zero at the top and bottom edge of the section to a maximum at the center.

- Bending Moment



Bending moments are internal forces that cause curvature in an attenuated member such as a beam or column. The induced curvature causes *lengthening* on the convex surface or edge (the bottom side of a simply supported beam) and *shortening* on the concave surface or edge. The lengthening is caused by tensile bending stress while the shortening is caused by compressive bending stress. This variation of stress over the cross section creates maximum stress at the top and bottom edges and zero stress at the centroid of the section (the midpoint of a symmetrical section). It is an important principle that the maximum shear stress and the maximum bending stress occur at different areas of the cross section of a member subjected to bending.

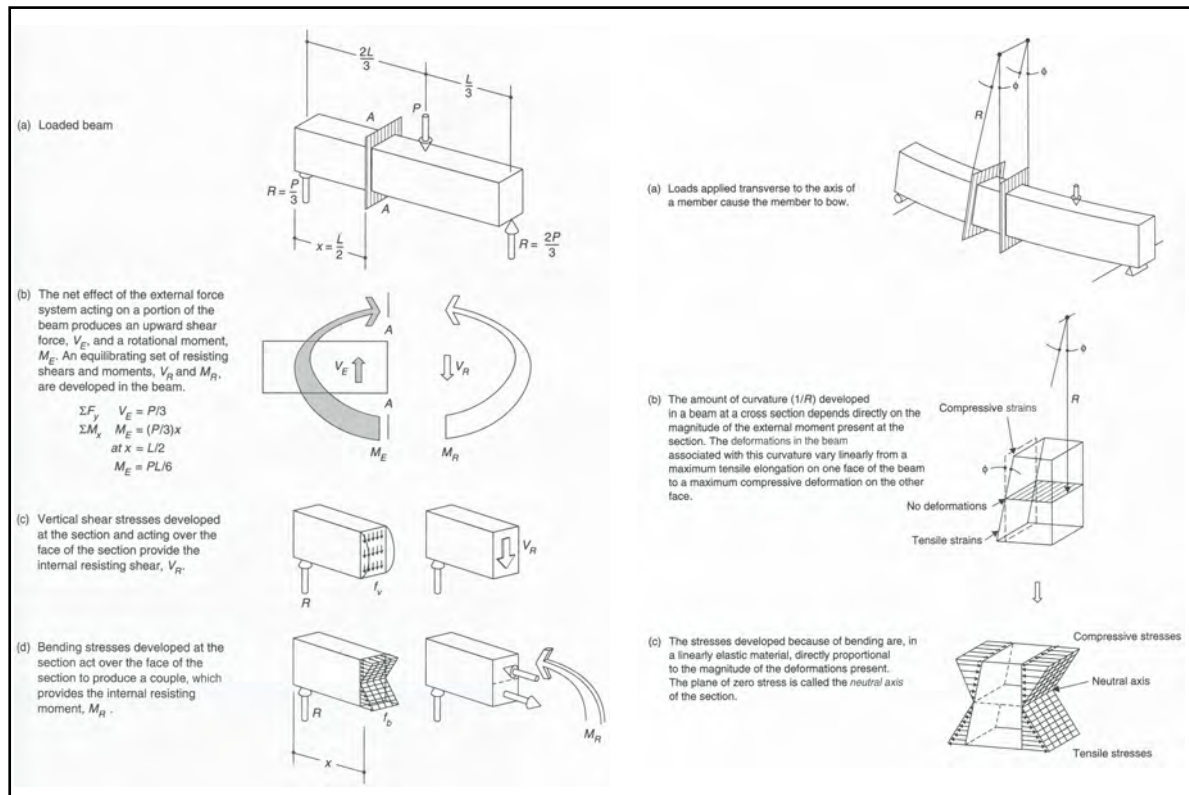


Figure B3.3: Bending Stress in a Beam. *Structures*, Schodek/Bechthold. Figure 6.4 p216

- Torsion

Torsion is created by a force acting on a member causing it to rotate about its longitudinal axis. Internal torsional forces generate a complex variation of stress on the member section. The result is a twisting deformation about the axis of the member. A common occurrence of torsion is in a girder that has a beam framing into one side of the girder. The load on the beam creates a rotational moment about the member axis. Closed hollow metal sections such as pipes and tubes are more efficient in resisting torsion.

### 3.2 Equilibrium and the concept of the free body

It is assumed that all structures such as those supporting buildings are not moving and therefore are in a state of equilibrium. Likewise, any portion of the building structure must also be in a state of equilibrium. In order to determine the type and magnitude of internal

forces in a structural member it is useful to imagine cutting a structural member into two parts and separating them. Each part is referred to as a *free-body*. To maintain the equilibrium of each part (that is to maintain a state of rest without translational movement or rotation) we imagine forces acting on each face of the cut section. In a two-dimensional statically determinate structure, these unknown forces can be found using equations of equilibrium such as  $\sum F_x = 0$ ,  $\sum F_y = 0$  and  $\sum M_o = 0$ .

### 3.3 Mechanical properties of materials

When materials are subjected to a stress they deform in a predictable manner. Tension causes elongation, compression causes shortening, etc. The amount of stress determines the amount of deformation. The precise amount of deformation is determined by the properties of the material, primarily the stiffness and elasticity.

Strain is a term that is used to describe the amount of elongation or shortening in a material subject to stress. It is defined as the ratio of the change in length of an element ( $\Delta L$ ) to its original length ( $L$ ), or  $\Delta L / L$ . This ratio known as strain, is represented by the symbol epsilon or  $\epsilon$ . There are no dimensional units to strain as it is a ratio or percentage. In the seventeenth century a scientist named Robert Hooke (1635-1703) discovered that for many materials there is a general relationship between stress and strain that is linear and constant. This relationship is known as Hooke's Law and states that for an elastic material, the ratio of the amount of stress present in a body to the amount of strain that is produced is a constant. This constant, which varies for different materials, is known as the modulus of elasticity,  $E = \text{stress} / \text{strain}$ . The larger the modulus of elasticity, the stiffer the material. The units of the Elastic Modulus are in  $\text{N/mm}^2$ . The modulus for ordinary steel is about  $205,000 \text{ N/mm}^2$  and for aluminum,  $69,000 \text{ N/mm}^2$ . Therefore, steel is about 3 times stiffer than aluminum.

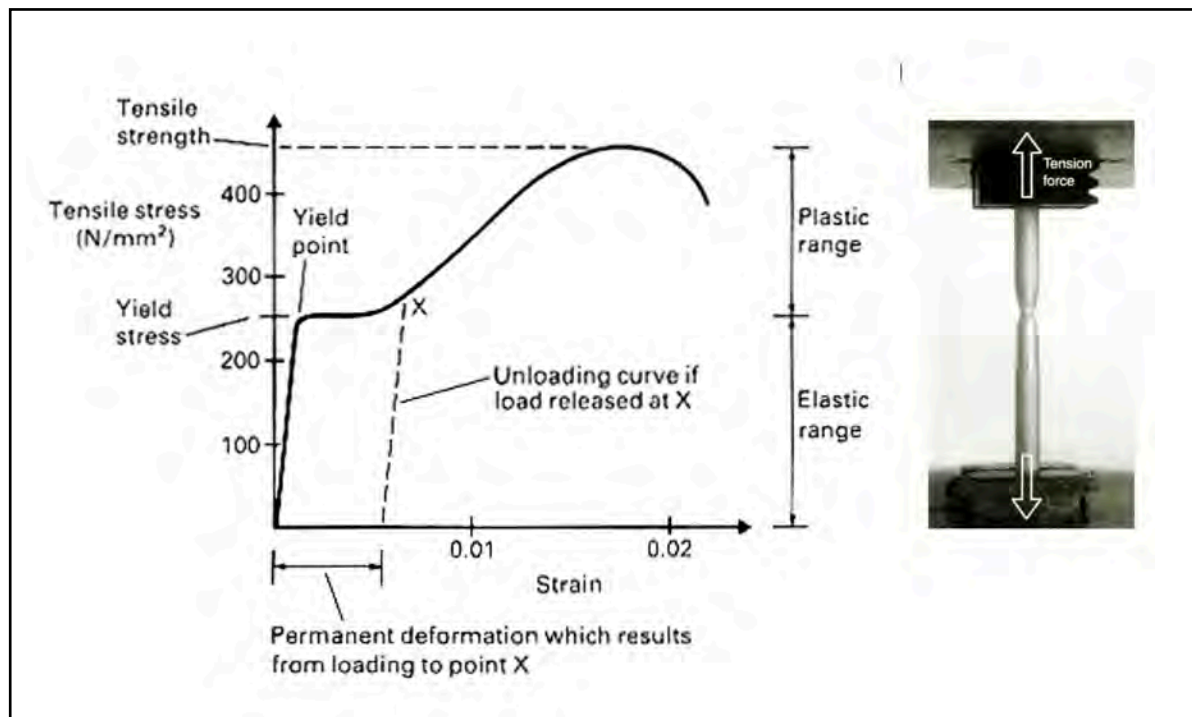


Figure B3.4: Stress | Strain Diagram for Steel

- Elasticity versus Plasticity

The property of a material that allows it to stretch or elongate under force and then return to its original shape when the force is removed is called elasticity. Past a certain point in the stress/strain test the specimen will continue to elongate or stretch under no appreciable additional load. This is referred to as the plastic range. Once in the plastic range the material undergoes a permanent deformation. For example, if the test specimen is unloaded and its length measured there will be a change: the length will have increased.

- Yield point

The point on the stress / strain curve at which the material yields and begins to stretch with no extra load applied. It marks the beginning of the plastic range. Not all materials have a yield point. Steel has a very distinct yielding point and this limit is used to determine safety margins in design under the working stress method of steel design.

- Strain hardening

Beyond the yield point the material stretches for a certain amount and then becomes stiff again. Further load may be applied and the material will offer resistance. This range of capacity beyond the yield point is known as the region of strain hardening.

- Ultimate Strength

A test specimen subjected to increasing load in the strain hardening range will offer resistance and continue to gradually elongate until the limit of the material's tensile strength is reached. At this point the specimen will abruptly break. Some necking or reduction of the cross sectional area occurs in the specimen which accounts for a lower stress before the material actually breaks.

- Ductility versus Brittleness

The property that allows a material to undergo large amounts of deformation before breaking is known as ductility. Rubber, for example, is extremely ductile. And so is steel although it is a much harder material. A material that can not stretch easily and breaks with very little elongation is called brittle. Glass is an example of a brittle material.

### 3.4 Shear Force and Bending Moment Diagrams

In order to better visualize the internal shear and moment forces on a structural member, we use graphical charts called shear and bending moment diagrams. For spanning structures the magnitude and distribution of bending moments is a critical concern. Knowing the shear force magnitude and location is also important, especially for steel reinforcement detailing in concrete structures. Also deep beam or plate structures require special consideration of shear. This is because in a deep beam, the internal shear forces at the ends of the beam often generate the critical stresses that dictate the beam design.

All spanning structures must resist an external applied moment force that is generated by the total load carried by the structure across a span. The method by which the span structure resists the applied moment varies. Beams develop an internal resisting moment force couple comprised of both tensile and compressive force resultants separated by a distance ("d") that acts as a moment arm. An arch develops the resisting moment couple with the horizontal reaction force at the springing point (base) and the horizontal component of the thrust line at a point along the arch. Each type of span structure has its own sectional configuration that determines the internal resisting moment.

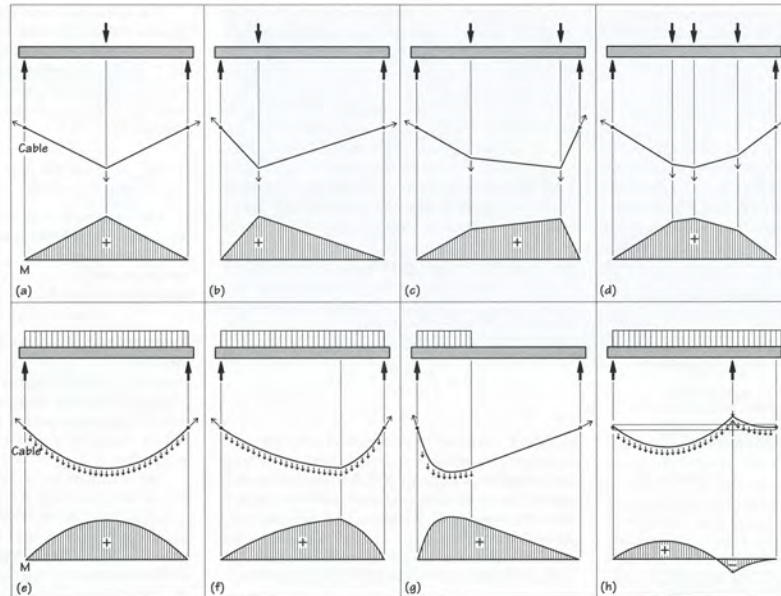
Structure	Basic Moment-carrying Mechanism	Free-body Diagrams with Respect to Rotational Forces		
		External Applied Moment	Part of Structure	Internal Resisting Moment
Truss				
Cable				
Arch				
Beam				
Plate on columns				
Plate on walls				
Dome				

Figure B3.5: Internal Resisting Force Couples for Span Structures.  
Structures, Schodek/Bechthold. Figure 13.4 p427

In order to draw (plot graphically) a shear or bending moment diagram it is necessary to determine the values of shear and moment forces at various points along the length of a structure. Usually a few points such as the maximum and minimum values are enough to determine the shape of the diagram. These can be obtained by cutting the structure at critical points (creating a free body diagram) and finding the unknown shear and moment forces with the use of the three equations of equilibrium ( $\sum F_x = 0$ ,  $\sum F_y = 0$ , and  $\sum M_o = 0$ ). There are certain visual aids (see Reference below) as well as relationships between load-shear-moment values that can also be useful. These are summarized in the Design Guide below.

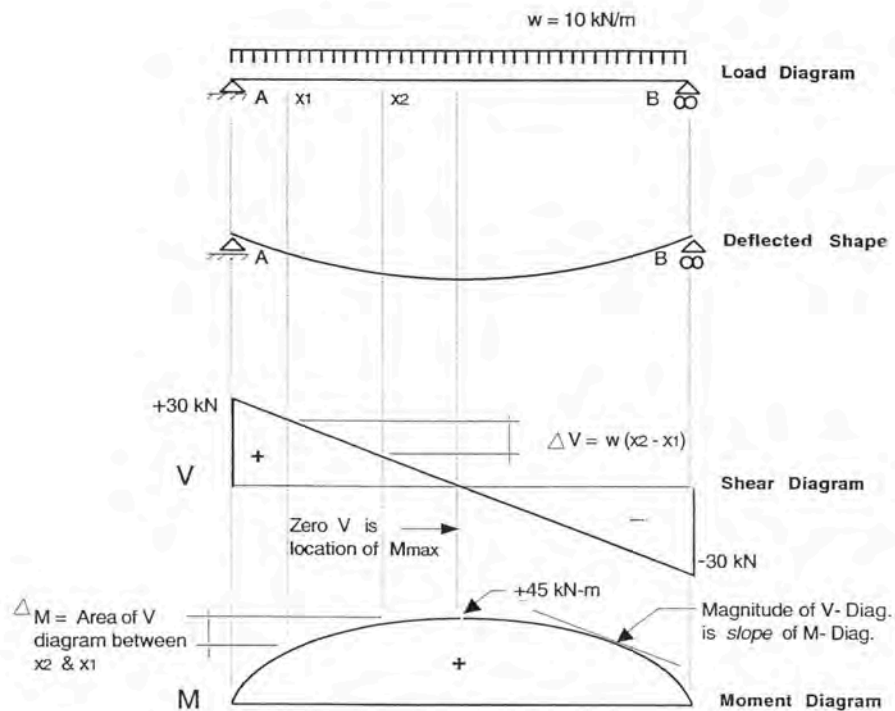
### Design Guide: Drawing the Shear and Moment Diagram

The Moment (M) diagram can in some cases be visualised by imagining a hanging chain suspended between two points (equivalent to the beam span) and supporting the same loads as the beam. The shape taken by the flexible hanging chain is an inversion of the shape of the moment diagram.



The shape of the moment diagram as the inversion of the hanging chain.  
*Form and Forces*, Allen/Zalewski. Figure 16.18 p443.

There are certain relationships between the Load, Shear and Moment diagrams that can be useful in constructing the V and M diagrams. These are summarised in the example below.





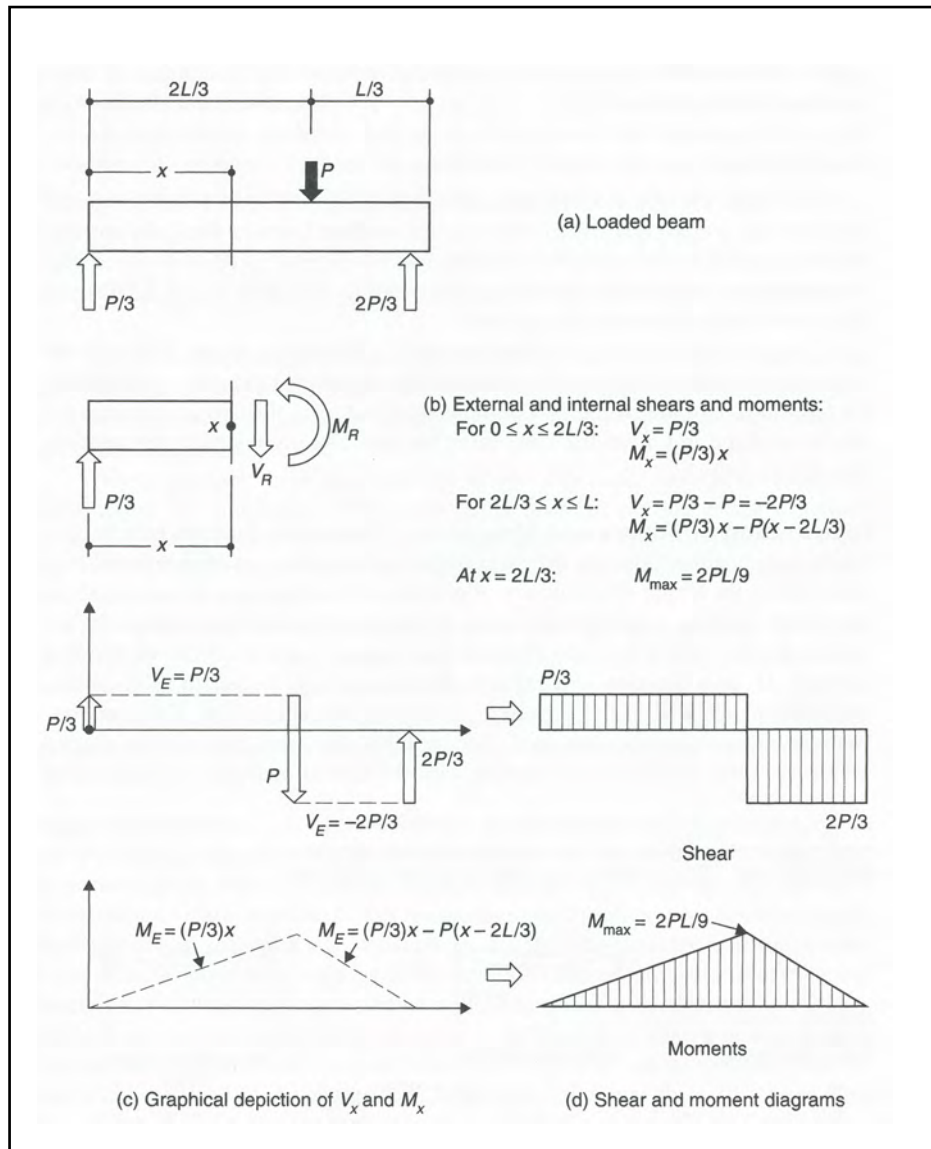


Figure B3.6: V-M Diagrams for a Beam. *Structures*, Schodek/Bechthold. Figure 2.41 p62

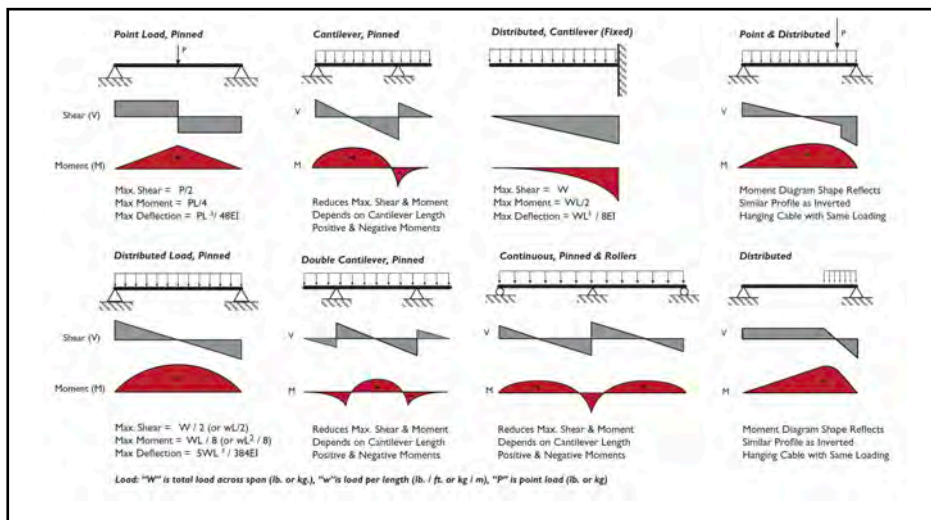


Figure B3.7: Typical V-M Diagrams. *Structures by Design*, R. Whitehead. Figure 3.0.32 p192

### 3.5 Member Stiffness: Moment of Inertia

The *Moment of Inertia* is a measure of a structural member's resistance to bending force. It is an abstract unit of measurement (N/mm<sup>4</sup>) based on the cross sectional area of the member and its configuration. For standard sections the value can be obtained in charts. For any rectangular section it can be easily calculated with the equation  $I = bh^3/12$  in which  $b$  is the width and  $h$  is the depth of the member.

The Moment of Inertia is used in the bending stress formula:  $f_b = Mc/I$  where  $f_b$  is the bending stress,  $M$  is the bending moment,  $c$  is the distance from the centroid of the section to the location of the bending stress value (Note: in a rectangular beam the value for  $c$  for the maximum bending stress is  $h/2$ ), and  $I$  is the moment of inertia. It is important to note that since the moment of inertia is in the denominator of the equation, an *increase* in the value of the moment of inertia results in a *decrease* in the magnitude of the bending stress.

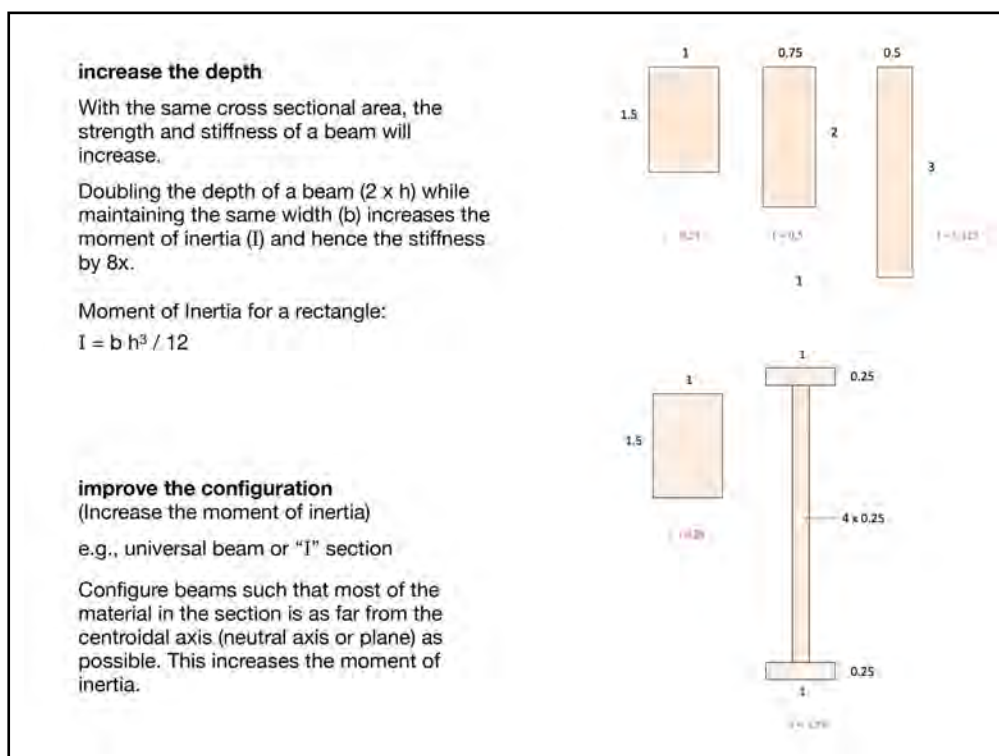


Figure B3.8: Improving Beam Efficiency

### 3.6 Member Stability: Resistance to Buckling

Axially loaded linear members, such as columns, will fail in one of two ways. Either the load on the member produces an axial compressive stress that exceeds the compressive strength of the material, and it fails by *crushing*. Or, if the member is relatively slender, and most columns are slender, the axial load on the member will cause it to bend in the middle and this bending will quickly lead to greater bending and member failure by *buckling*. This is a type of instability failure.

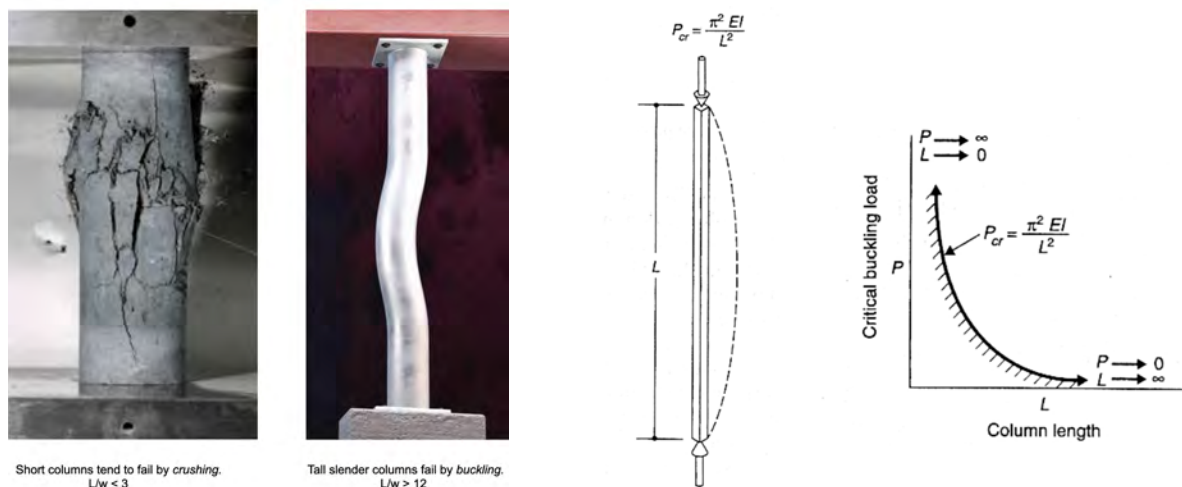
The phenomena of buckling failure is fairly complex and it causes columns and other attenuated compressive elements like struts and truss compression members to fail at loads well below the allowable stress level of the material due to compressive stress alone ( $f_c < P/A$ ). The challenge of predicting what load would cause a member to fail in buckling was solved by the mathematician Leonhard Euler around 1759. He derived an equation

from the testing of long, slender members that correctly indicated the failure load based on the physical properties of the member and applied loading. The equation which still forms the basis for column load tables today is:

$$P_{cr} = \pi^2 EI / L^2$$

- $P_{cr}$  is the critical load causing the column to buckle,
- $E$  is the elastic modulus of the material that indicates the stiffness of the material,
- $I$  is the moment of inertia that accounts for the configuration of the column in either the x or y direction of bending, and
- $L$  is the length of the column.

The tendency of a member to buckle in one direction or another can be overcome by equalizing the moment of inertia in the x and y directions or by using bracing (most effective at mid-length) in the weak direction (direction with the smaller moment of inertia).



Two modes of column failure: crushing or buckling.

Euler's equation for column buckling.

Figure B3.9: Modes of Column Failure. *Structures*, Schodek/Bechthold. Figure 7.3 p280

## REVIEW QUESTIONS

- Describe how a load generates member forces and stresses on a structural element (e.g., a beam).
- How can the configuration of a structural member section improve the efficiency of that member?
- How does the material of a structural member impact member stiffness and deflection?
- Draw/sketch V and M diagrams for a few different types of beams: i) simply supported beam with both distributed load and a concentrated load at midspan. ii) a propped cantilever beam with distributed load on the main span and a concentrated load at the end of the cantilever. iii) a two span continuous beam with distributed load on one span and a concentrated load at the midspan of the other.

## SELECTED REFERENCE

- 1) *Structures*, Daniel L. Schodek and Martin Bechthold, 2014, Pearson.
- 2) *Structure and Architecture*, Angus J. Macdonald, 2001 2nd Ed. Architectural Press.
- 3) *Structures by Design*, Rob Whitehead, 2020, Routledge.
- 4) *Form and Forces*, Edward Allen and Wacław Zalewski, 2010, John Wiley & Sons.

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## Section C Structural Materials and Construction

### INTRODUCTION

Materials that are used in modern construction for structure have been introduced gradually over time. First wood and clay bricks, then stone. The first use of steel in the form of wrought iron occurred late in the 18<sup>th</sup> c. Industrialization and economic growth in the nineteenth century saw the mass production of steel and the beginning of standardized structural sections. Reinforced concrete introduced in the early 20c has dominated the construction industry in many parts of the world since. Local availability of materials (sand and gravel are common almost everywhere while cement and steel reinforcement are easily transported), labor intensive and low-tech construction practices make it adaptable to many regions.

Aluminum was introduced as a structural material in the mid-20c. It is the second most produced metal after iron. Other materials that have gained structural application include: frp (fiber reinforced plastic), pvc (polyvinyl chloride), glass (tempered), carbon fiber, and soil (rammed earth).

### TOPICS

#### 1.0 Construction and the Environment

Today we are aware of the consequences of energy consumption on global warming and on related issues of sustainability. In advanced developed countries the largest consumer of energy is the built environment. In Hong Kong it is reported that buildings use up to 90% of total electricity consumption (Note that as of 2023 <1% of electricity in Hong Kong is generated from non-fossil fuel energy sources).

*"Building Energy Efficiency: The Key to a Green City"* at [www.hongkong.ahk.de/news](http://www.hongkong.ahk.de/news)

A large portion of life cycle GHG (greenhouse gas) emissions of an office building are attributed to operational carbon (67%) as opposed to embodied carbon (27%). Operational carbon, that is energy consumption of fossil fuel generated electricity, is in lighting, ventilation, air conditioning and other uses. Embodied carbon is the material fabric of the building.

1.1 Embodied energy is the total energy required for the extraction, processing, manufacture and delivery of building materials to the building site. It does not include the energy consumed for the placement or disposal of materials.

In addition to the consideration of the amount of embodied energy of various materials, the selection of materials based on energy consumption should also take into account:

- Durability of materials (long life)
- Use of locally sourced materials (minimize energy cost of transportation)
- Use of recycled materials
- Specifying standard sizes of materials (reduces waste)
- Use of materials manufactured using renewable energy sources (human labor, wind/ solar power, etc.)
- Consideration of ease of disassembly for purpose of recycling and disposal



Figure C1.1 Embodied energy of various construction materials.

1.2 Material or resource depletion is an important issue in our time. The extraction and processing of certain materials for use in construction (e.g., cement for concrete) requires large amounts of energy and produces waste products that are harmful to the environment. Some materials are limited in existence and may be depleted in the future. Materials which are renewable (e.g., engineered wood) are also referred to as *sustainable*.

1.3 The amount and type of materials consumed in a project can be controlled through design. Buildings today weigh less than ever before. One reason is the use of lighter but stronger materials. Post-tensioned floor plate design reduces the volume of concrete required. Aluminum fenestration is significantly lighter than steel although it has the highest embodied energy of any construction material. The increasing use of engineered lumber is reducing overall weight. The ongoing trend towards lighter construction with less material also reduces the size of foundations.

A second trend in reducing weight is through more efficient structural design. Efficiency of structure (the ability to carry greater loads with less material) however, does not always correlate to a more economical solution. The fabrication of a steel truss is probably more costly than the use of a reinforced concrete beam for a given span, even though the savings in weight (as well as embodied energy) would make it a more sustainable choice.

Nonetheless, there are many cost effective design strategies to reduce weight by increasing the efficiency of member design. (See section 2.3 in Part A of the Study Guide.)

## 2.0 Construction Systems

There are a few basic construction systems that dominate building design today. By far, reinforced concrete framing is the most common. The other systems include steel framing, wood framing, and masonry bearing wall construction. Stone is a structural material that was used extensively in the past for structure but today is mainly a non-structural cladding material.

### 2.1 Concrete

2.1.0 Concrete was invented by the Romans who recognized its plastic, formable quality and used it extensively to create an architecture of walls, arches, vaults and domes. In the 19c an understanding of how to use steel reinforcement to compensate for the low tensile strength of concrete made possible the invention of a new composite material, reinforced concrete. Initially, this led to the development of the system *Hennebique* (1892), a *monolithic* frame construction employing columns, beams and slabs formed on site (*in situ*). Today, factory production of concrete elements (precast concrete) is common.



2.1.1 Concrete is composed of (by volume approximately): cement (11%), water (16%), air (6% ), sand (26%) and coarse aggregate (41%). The exact proportions of these constituent elements will vary depending on various performance requirements. Mix proportions are specified in section 4.2.6 of the Code of Practice for Structural Use of Concrete (COP-SC). The *quality* of the various materials must conform to parameters as specified in the Building Regulations of Hong Kong. These are also described in the COP-SC.

The strength of concrete is determined by the *grade of concrete* and is found in Table 3.1 (COP-SC). Strengths vary from a grade of C20 (20 N/mm<sup>2</sup>) to C100 (100 N/mm<sup>2</sup>). The full designated strength of concrete is only obtained after a period of time (28 days) and under controlled conditions to ensure that the concrete is cured properly. For example, after pouring, concrete must be protected from heat to prevent loss of moisture due to evaporation. In some climates concrete must also be protected from extreme cold temperature. Samples of concrete for testing (Concrete Cubes) and other quality assurance controls are outlined in sections 10 and 11 of the COP-SC. These include: strength tests, placement and compacting of concrete, curing time periods and protection of surfaces, construction and removal of formwork and falsework, surface finishes, number and quality of construction joints, and reinforcement and prestressing steel.

2.1.2 An architect visiting a construction site with in-situ concrete formwork in process should be able to comprehend the layout and structural role of the reinforcement steel as it's being put in place. Basically in concrete framing we need tension steel rebar wherever the bending stress is in tension, and some vertical steel (stirrups) in areas where the shear force is high (generally near the supports). The position of the tension steel rebar depends on the sign of the bending moment in the beam (positive/negative) which in turn is related to the deflected shape of the beam or column. The tension steel will always be placed on the outside of the curvature. In areas of exceedingly high compression, reinforcing steel may also be used. This is referred to as *compression steel*.

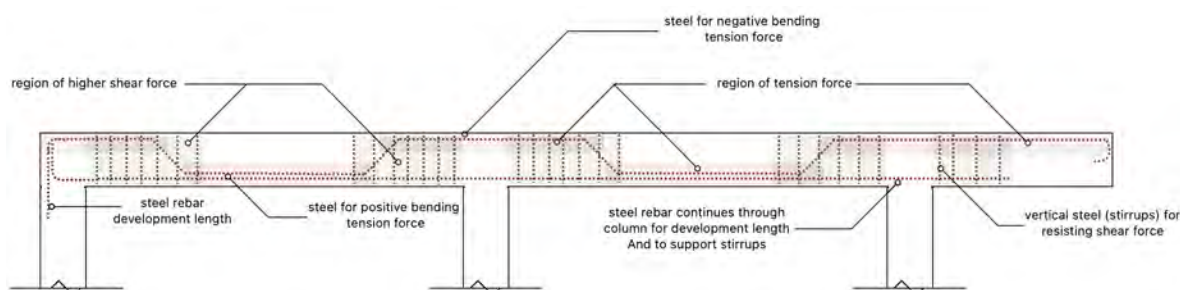


Figure C2.1: Steel reinforcement in a continuous concrete beam.

In concrete slabs with two-way bending, steel reinforcement is provided wherever bending moments cause tension stress. Bars are configured in *bands* with two bands centered on the columns in each direction (*column strips*) positioning the steel in areas of tension (top over the columns and bottom in mid-span). Bands are also required in each direction for the *middle strips* with the steel placed at the bottom. For one-way slabs the dominant steel reinforcement will be in the direction of the span with *temperature* or *shrinkage reinforcement* in the perpendicular direction. This can be placed in the middle of the depth of the slab.

### 2.1.3 Prestressed Concrete

Prestressed concrete uses *prestressing* or *post-tensioning* to achieve additional strength and limit deflection. In prestressing, a concrete beam is formed around a stretched (pretensioned) high-strength steel wire or tendon (A & B). After the concrete is cured, the

force on the steel tendon is released and through the bond of the concrete to the steel, the force is transferred into the cross section of the beam (C). The tendons are generally located below the center of gravity of the section so that the greater compressive force is transferred to the region of the beam where the tension force will later develop (at the bottom in a simply supported beam). This has the tendency of inducing some upward curvature to the beam. Upon placement of imposed (live) loading the beam will straighten some thus minimizing the amount of deflection mid-span (D). The live load will create an additional bending stress that is symmetrical (equal tension and compression). When added to the existing stress in the beam the compression stress at the top will increase and the compression stress at the bottom will decrease to either zero or a small amount of tension. This process is carried out in a plant.

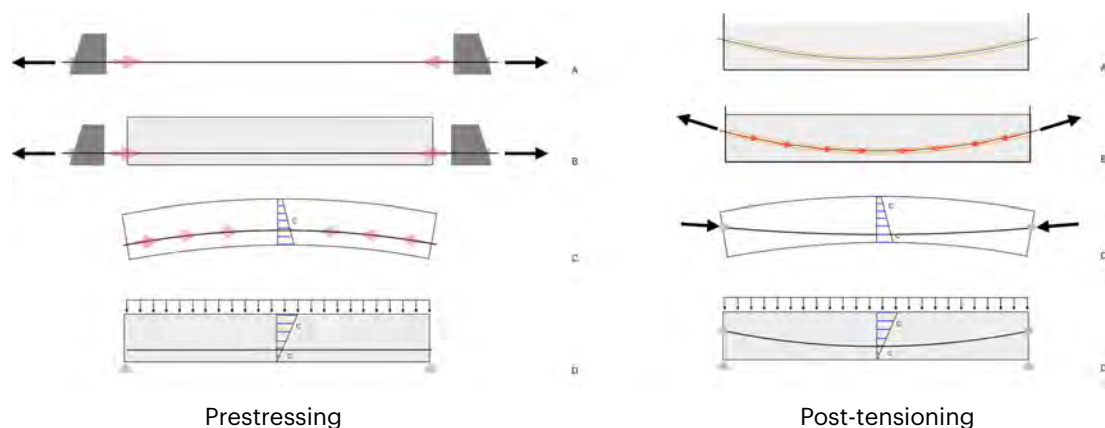


Figure C2.2: Prestressing

In post-tensioning, the concrete elements to be post-tensioned have tubes cast in place into which steel prestressing tendons are introduced (A). After the concrete is cured, the tendons are stretched using a jacking device (B) and the ends of the tendons are clamped against the outside of the end of the beam or slab (C). This induces a large compression force on the ends of the structural member (the tendons are not bonded with the concrete as they are protected by the tubes) that creates compression force on the section throughout and an upward curvature of the member. Finally with the addition of live loads, the stress on the section will be transformed in the same way as with prestressing and the upward camber of the member will flatten. The post-tensioning procedure is usually done on the site.

2.1.4 Precast concrete is the production of both standardized and custom designed transportable structural components in a factory. They are manufactured with both normal density and lightweight concrete and can be prestressed for improved efficiency. Factory control produces elements with greater consistency of strength, durability, material finish and dimensional precision. Precast concrete tends to be modular; the cost effectiveness is increased by the reuse of casting forms.

Precast structural elements are generally one-way spanning structures, They require careful attention to connections. They lack the rigidity of monolithic poured in-place concrete. Buildings constructed of mostly precast components require carefully designed lateral bracing systems. *(For further study refer to the Code of Practice for Precast Construction, 2016.)*

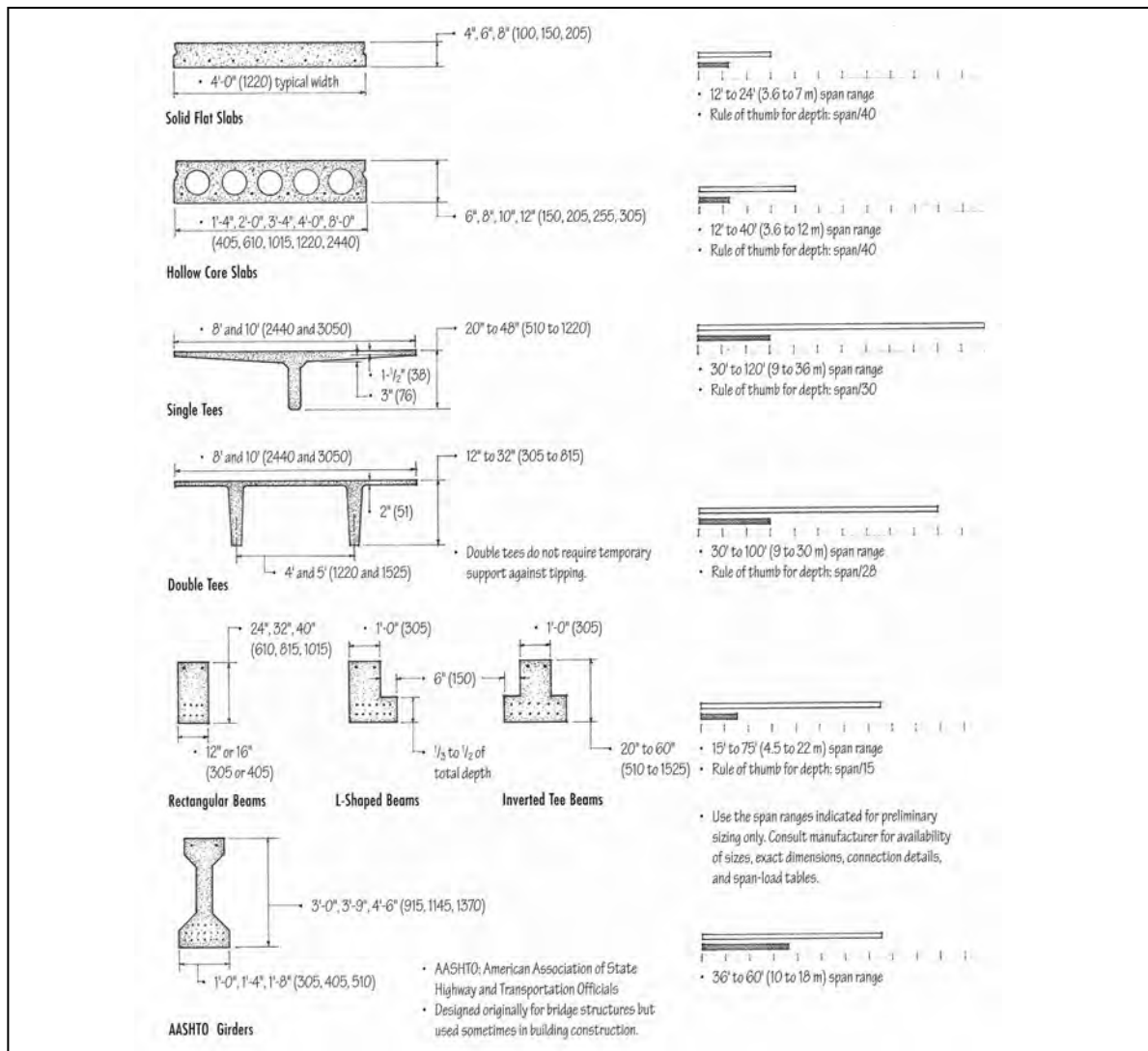


Figure C2.3: Types of Precast Concrete Units. *Building Construction Illustrated*, Ching/Adams. p4.12

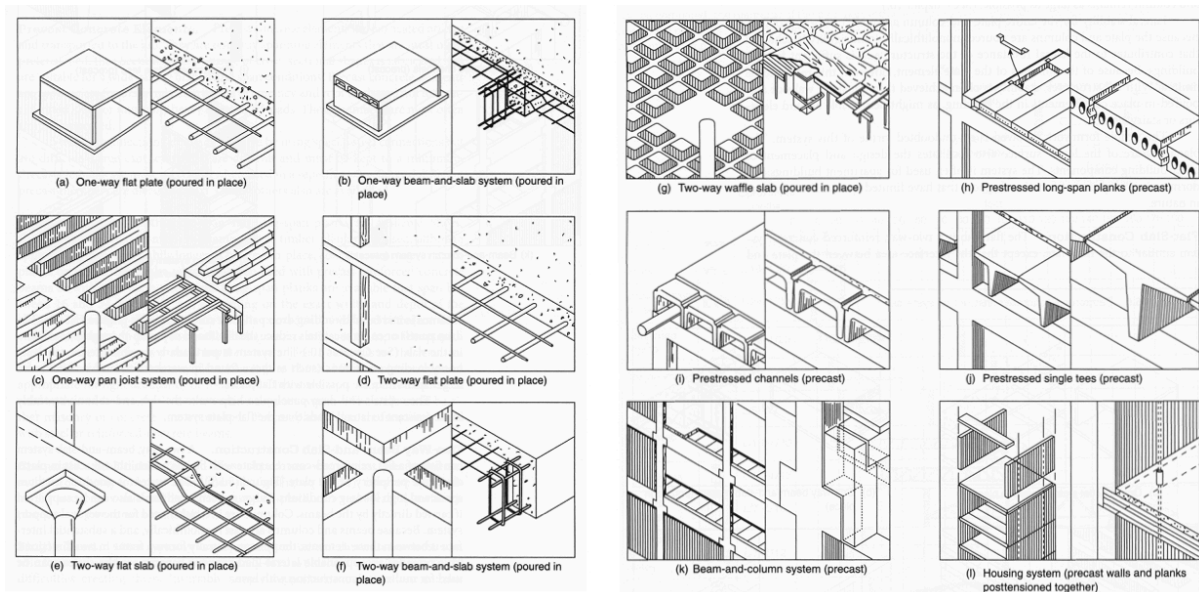


Figure C2.4: Types of Concrete Construction. *Structures*, Schodek/Bechthold. Figure 15.5 pp491-2

## REVIEW QUESTIONS

- What are some of the ways we can reduce energy consumption in the design of buildings?
- What are the main constituents of concrete? What component has the most adverse effect on strength?
- How is concrete that's been placed (already poured) determined to have sufficient compressive strength?
- What is the compressive strength of the highest grade of concrete? The lowest?
- Describe the positioning of steel rebar in a continuous concrete beam.
- How soon after pouring concrete can the shuttering (formwork support) be removed for a flat slab? For a cantilevered balcony?
- According to the Code of Practice, what kind of water may be used in a concrete mix?
- What is the likely outcome if we use less cement than specified in a concrete mix?
- Describe the procedures for both prestressing and post-tensioning. What are some differences?
- What are some advantages of precast concrete? What are some disadvantages?

## 2.2 Steel

2.2.0 Steel, which is essentially *iron* has been known to man for thousands of years. However, only in the last two hundred years has it emerged as a major structural material. Today it is the most widely used material in construction, both as steel framing and as reinforcement in concrete construction.

2.2.1 Iron becomes steel when the excess carbon is removed in the smelting process. Iron can have up to as much as 6.7% carbon. The normal range for cast iron is 2.1 - 4%. Below 2.1% carbon (by weight) is steel. Carbon makes iron brittle so removing the carbon from iron gives steel ductility. However some carbon content allows steel to be hardened and made tougher with increased strength. *Wrought iron* was the forerunner of steel. With a carbon content below .08% it is equivalent to modern day mild steel. Wrought iron was not easily produced but was a sought after material because of its toughness, ductility, corrosion resistance (iron easily rusts) and malleability. Early iron construction used wrought iron before steel was mass produced economically through the Bessemer Converter process in the mid-19c.

Steel is combined with other elements to produce various *alloys* that have special characteristics. For example, when combined with up to 20% chromium and nickel the alloy is called *stainless steel*. It is highly corrosion resistant but also much harder and more brittle than carbon steel. And it is very costly to produce. There are many alloys that are produced and have varying properties and particular applications.

Structural steel is produced in steel mills by melting steel *ingots* (processed iron) and then extruding the heat-softened metal into various standardized shapes (e.g., universal beam sections, angles, pipes, etc.). These sections are very precise and are used for steel construction worldwide. Plates of steel are also produced for a cold rolling process that produces thin and even surfaces that can be further worked into various sectional shapes (e.g., steel decking).

Steel can also be cast into almost any shape. Since it is a strong, isotropic material, it is ideal for complex shapes that require high strength with ductility. It is commonly used for complex, three-dimensional connections.

### 2.2.2 Structural Properties

Steel is produced in various grades of strength. The most common grades of structural steel are designated as S235, S275 and S355. These are grades supplied in accordance with BS (British Steel) and EN (European Standard 10025). The number “235” in the designation is the minimum yield strength of the steel for thickness not exceeding 16mm (for larger thicknesses the yield strength decreases slightly).

Steel has a Young's Modulus (E) of 205,000N/mm<sup>2</sup>. This is the ratio between stress and strain and indicates that steel is very stiff (it has the highest E value of structural materials). Steel is also a *ductile* material; a sample of steel to which a force is applied, will stretch a certain amount and then return to its original length after the stretching force is removed. If the sample continues to be stretched with additional force, the *yield point* is reached (for S235 it is 235N/mm<sup>2</sup>) and the steel enters the *plastic* zone in which it stretches with very little increase in force until it reaches the *strain hardening phase* at which the steel becomes stiff again (but with a large degree of permanent deformation or elongation that is not recoverable). With more force applied, the steel will continue to stretch until it reaches its limit (*ultimate tensile stress*) and breaks. (Refer to Figure B3.4 in part 3.3 of Section B, pp39-40)

Steel is a near perfect isotropic material (also aluminum and glass). It has identical properties in all directions. This greatly simplifies the calculation of stress and allows steel to be cast into complex three dimensional shapes.

Advantages	Disadvantages
<ul style="list-style-type: none"><li>• High strength to weight ratio.</li></ul>	<ul style="list-style-type: none"><li>• Toughness makes it difficult to work and shape.</li></ul>
<ul style="list-style-type: none"><li>• Quality control of the production of materials: dimensional precision of components.</li></ul>	<ul style="list-style-type: none"><li>• High density (weight) of steel makes handling difficult. Cranes required for construction.</li></ul>
<ul style="list-style-type: none"><li>• Distinctive appearance: smooth, sharp edges.</li></ul>	<ul style="list-style-type: none"><li>• Oxidation (rust) requires protection.</li></ul>
<ul style="list-style-type: none"><li>• Off-site prefabrication provides fast erection.</li></ul>	<ul style="list-style-type: none"><li>• Fire protection of steel required.</li></ul>
<ul style="list-style-type: none"><li>• Slenderness and longer span of steel framing facilitates open planning and accommodation of services</li></ul>	<ul style="list-style-type: none"><li>• High cost compared to timber or r/c.</li></ul>

Figure C2.5: Advantages and Disadvantages of Steel.

2.2.3 Structural steel framing uses various steel elements (hot-rolled steel beams and columns, open-web joists, metal decking, etc.) to form assemblies connected together by bolting or welding. Steel framing generally lends itself to orthogonal grid configurations but can be adapted to radial and even free-form arrangements. The overall stability of steel frame structures, especially those that are primarily pin-connected assemblies, needs to be considered carefully. Stability can be achieved in various directions by either moment resisting frame action (fixed rigid connections) or bracing (lift core, shear wall or diagonal struts/cables). Horizontal diaphragms (rigid floor/roof construction) are also required.



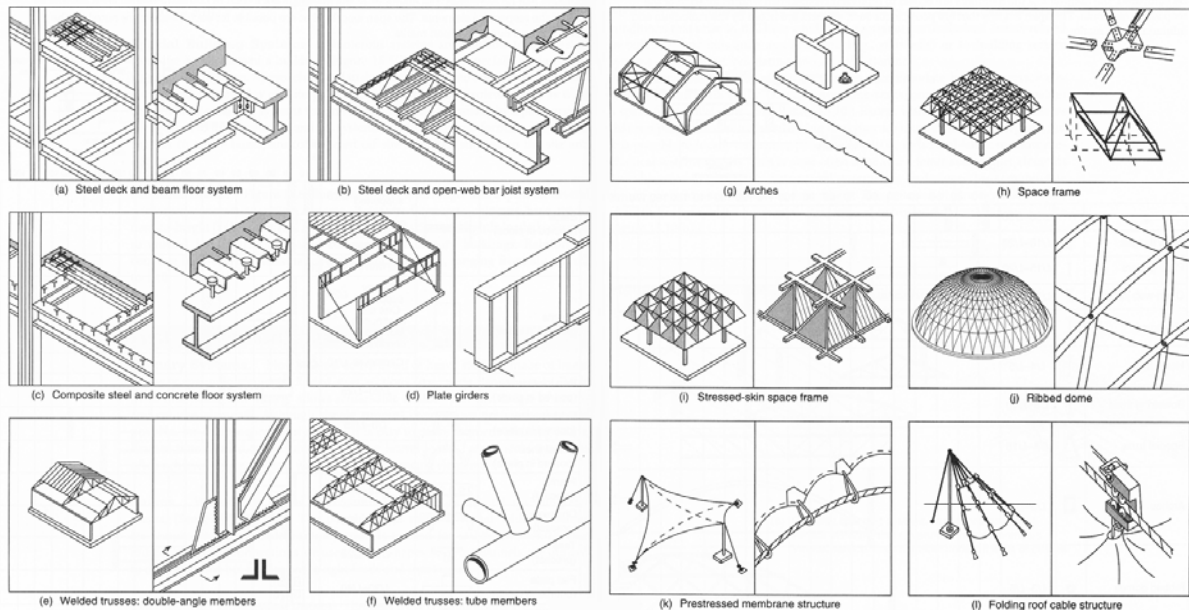


Figure C2.6: Types of Steel Construction. *Structures*, Schodek/Bechthold. Figure 15.7 pp496-7

## REVIEW QUESTIONS

- Steel is an isotropic material. What does this mean structurally?
- What important property of steel remains constant even as the strength of different grades of steel varies?
- Name two reasons why exposed steel needs to be protected.
- Steel is 3x heavier than aluminum. What other property of steel is 3x greater?
- Excess carbon in steel is an advantage or disadvantage? Explain.

## 2.3 Glass

2.3.0 Glass has existed in some form for more than 5,000 years, however, true transparent glass appeared only in the 15c. With the invention of *float glass* (Pilkington ca1957), large, high quality distortion-free flat sheets of glass became available and led to the revolution in curtain wall design. In recent times, structural glass has found new applications for horizontal support (beams), glass fin bracing, handrails and balustrades in addition to fenestration.

2.3.1 “Glass is a brittle material and normally fails in a sudden and catastrophic manner. Glass is weak in tension because of the nature of its atomic structure and the resulting susceptibility to flaws and inability to resist crack propagation. When glass is loaded in tension it behaves purely elastically until it fails suddenly at the ultimate tensile strength, unlike structural steels, aluminum alloys and even reinforced concrete that can accommodate plastic deformation after yield point.” (Reference: COP-G 2018. See especially Figure 4.1 Stress-Strain Graph of Steel, Aluminium and Glass.)

2.3.2 Because of its brittleness, glass has had limited application as spanning structure. The difficulties encountered by glass connections (susceptibility to stress concentrations) are also a factor. Nonetheless, thin glass plates used as horizontal fenestration (skylights, roof glazing) span distances that are large in relation to their depth or thickness.

The majority of glass in buildings is fenestration in a vertical orientation and thus subject to wind loading perpendicular to its surface. The thickness of glass will vary from 6mm to

25mm (nominal) according to the size of the glass (area) and the wind load. The support of the glass either in a frame or with the use of point support such as spider fixing must be carefully designed. Wind pressures on glass, especially in tall buildings, must consider suction or negative pressure. A full-scale mock-up test to verify performance is recommended.

2.3.3 The various types of glass produced include: *annealed* (ordinary window glass), *heat-treated*, *heat-strengthened*, *tempered* (safety glass), and *laminated* (polyvinyl butyral or PVB as interlayer). For safety reasons, most glass used in situations where breakage could lead to injury (especially hi-rise glazing) is tempered glass. As a result of its treatment, tempered glass has a bending strength about four times that of annealed glass. Its special characteristic, however, is that if it breaks it produces small square-shaped granules as opposed to sharp-edged shards.

Laminated glass is also a type of safety glass in that the interlayer will prevent glass shards from being dislodged. The PVB layer is also a barrier for the transmission of sound. Laminated glass can be produced with interlayers that reduce ultraviolet radiation. Multiple layers of laminated glass can be bullet and blast resistant.

In summary, glass can be produced with many different physical and performance characteristics. Selection of the appropriate type of glass will balance the cost and the specific task required.



Figure C2.7: Structural Glass. Examples.

## REVIEW QUESTIONS

- What type of glass would be used for a glass door in a commercial venue?
- What is safety glass?
- Is glass an elastic material like steel or aluminum?
- What is the rationale behind the spider connective device? Describe the transmission of forces in a spider fixing.

## 2.4 Wood

2.4.0 A naturally occurring material, wood or *timber* is stick-like due to its origins, and after processing (seasoning and cutting), *lumber* is structurally most suitable for frames. Surfaces such as roofs and floors can be made from elements of lumber by framing, a system of linear trimmed pieces of wood that consist of columns, beams, and floor planks. Later *plywood*, an early engineered lumber and a composite material of thin *veneers* of wood peeled off logs and then glued into laminations to create flat sheets of various thicknesses, replaced planks as a more cost effective surfacing material. Today there is a growing acceptance of engineered wood construction as it is probably the most sustainable building material in use.

2.4.1 Wood is a cellular organic material that is a natural composite of *cellulose* fibers that are strong in tension, held together by *lignin* that resists compression. Because of the cellular structure of wood, it is an *anisotropic* material, strongest in the longitudinal direction parallel to the grain. It has a high strength to weight ratio and is equally strong in both tension and compression, which is favorable for bending. The strength of wood reaches an optimum value at about 12% moisture content (unseasoned or green timber is about half the compressive strength of seasoned lumber). In general the density of wood correlates with its strength; the heavier wood is harder and stronger.

Wood is an easy material to work with due to its density (approximately 1/12<sup>th</sup> - 1/15<sup>th</sup> that of steel). It can be protected with finishes that can also enhance its natural beauty.

Some of the disadvantages of wood are its susceptibility to decay (both moisture rot and insect), its flammable nature, the limitation of the length of lumber (trees are only so big) which highlights one of wood's important constraints; connections. Wood connectors range from small nails, nail plates and screws to more complex internal devices like *split-ring connectors* and *cast steel nodes*. Various types of metal plates with bolts are also commonly used for large members.

#### 2.4.2 Why is wood a sustainable material?

- Fast growth trees are cultivated for wood fiber that is used to produce a number of engineered wood products such as *laminated strand lumber* (LSL), *oriented strand lumber* (OSL), and *parallel strand lumber* (PSL). Also smaller and younger trees produce smaller size pieces of wood that can be cut and joined by glue (*glue-laminated wood*).
- Using wood reduces the CO<sub>2</sub> footprint of a building. It has half the embodied energy of steel and it doesn't produce the large amount of CO<sub>2</sub> emissions created in the production of concrete. Also trees remove CO<sub>2</sub> from the air and store the carbon in wood.
- Trees are plentiful throughout the world (or should be) so harvesting and processing can be done locally minimizing energy consumption through transportation.
- Wood is a material that lends itself to off-site prefabrication that reduces waste and improves material efficiency.
- Recycling. Wood is relatively easier to recycle, either by reclaiming old lumber for reuse or by shredding discarded wood for use in engineered wood products. It can also be disposed of through biological decomposition or used as a fuel (although this method is responsible for a significant amount of green house gas).
- Overall, wood construction is lighter than concrete or steel for comparable building size and therefore requires less foundation structure.

2.4.3 Presently, the majority of wood used in construction is in the form of *dimensioned lumber*; timber that is seasoned and cut in factory controlled conditions into precisely dimensioned and standardized lumber elements (2x4, 2x6, etc. in inches and 50x100, 50x200, etc. in metric). These boards are then cut to required length and used independently as framing members (purlins, rafters, beams, etc.) or combined in various ways to produce manufactured wood components such as I-joists, trusses, etc. They can also be easily combined to produce stronger, *built-up* elements such as multiple-shaft columns and beams with spacer blocks.

Another important wood product is the panel. Panels are prefabricated to standard sheet size (4' x 8' or 1200mm x 2400mm) and are made of various wood materials. *Plywood* is made from thin layers of veneer glued together with the grain rotated 90° on each alternate layer for uniform non-directional strength. *Composite panels* have outer layers of veneer with an interior core of reconstituted wood fibers. *Oriented strand board* uses long shreds (strands) of wood compressed and glued into layers, with the orientation of the strands or grain rotated similar to plywood. *Particle board* uses finer wood particles (saw

dust) that are compressed and bonded into a panel, that may receive an outer finish layer of veneer or plastic laminate. *Fiberboard* uses the most fine-grain wood particles that produces a more stable and stiffer panel than particle board. *Medium density fiberboard* (MDF) is the most common form.

The two most important products that use small-size wood pieces from fast growth trees are *glu-lam* and *cross laminated timber* (CLT). Glu-lam structural elements such as beams, arches and frames are well established products in construction. Glu-lams have several important advantages over lumber. First, since they are built-up from small wood pieces, the length of the member is unlimited and can be formed into many shapes. Curves and tapered profiles are not difficult to produce. A second advantage is that the individual pieces of wood can be selected on the basis of their strength and placed where higher strength is more beneficial in the section. This uses the different grades of lumber more efficiently. Cross laminated timber is composed of layers of small pieces of trimmed lumber glued parallel to each other, and with each layer rotated in a manner similar to plywood. This produces thicknesses and panel sizes comparable to precast concrete slabs that can be used for floors, roofs and load bearing walls.

Type	Product	Forming methods
Sawn	dimensional lumber log, squared timber, plank, board batten	chainsaw, band saw, circular saw
Laminated	plywood	rotary cutting of log + adhesive
	laminated veneer lumber (LVL)	rotary cutting of log + adhesive
	laminated strand lumber (LSL & PSL)	sliced veneer strands + adhesive
	glue laminated lumber (Glu-lam)	squared lumber + adhesive
	cross laminated lumber (CLL)	squared lumber + adhesive
Pressed	oriented strand board (OSB)	compressed strands/flakes + adhesive
	particle board	compressed waste wood chips + adhesive
	medium-density fiberboard	pressure formed wood fibers
	masonite	pressure formed wood fibers

Figure C2.8: Types of wood used in construction.

2.4.4 Prior to the use of modern steel connectors, wood post and beam frames were held together by various *interlocking* joinery such as mortise and tenon, dovetail and box joints. For light framing and roofing, nails were adequate. For larger pieces of wood, various types of bolted connections were devised. Plate element connectors were devised to help transfer and distribute the shear forces. The use of an intermediary element of steel simplified certain types of member joining, especially in trusses, enabling the transfer of axial forces without the development of bending resistance. Many structures today employ complex three dimensional cast steel joints as connectors, mimicking the joints of the human skeleton in their elegant form.



Figure C2.9: Wood Structures. Examples.



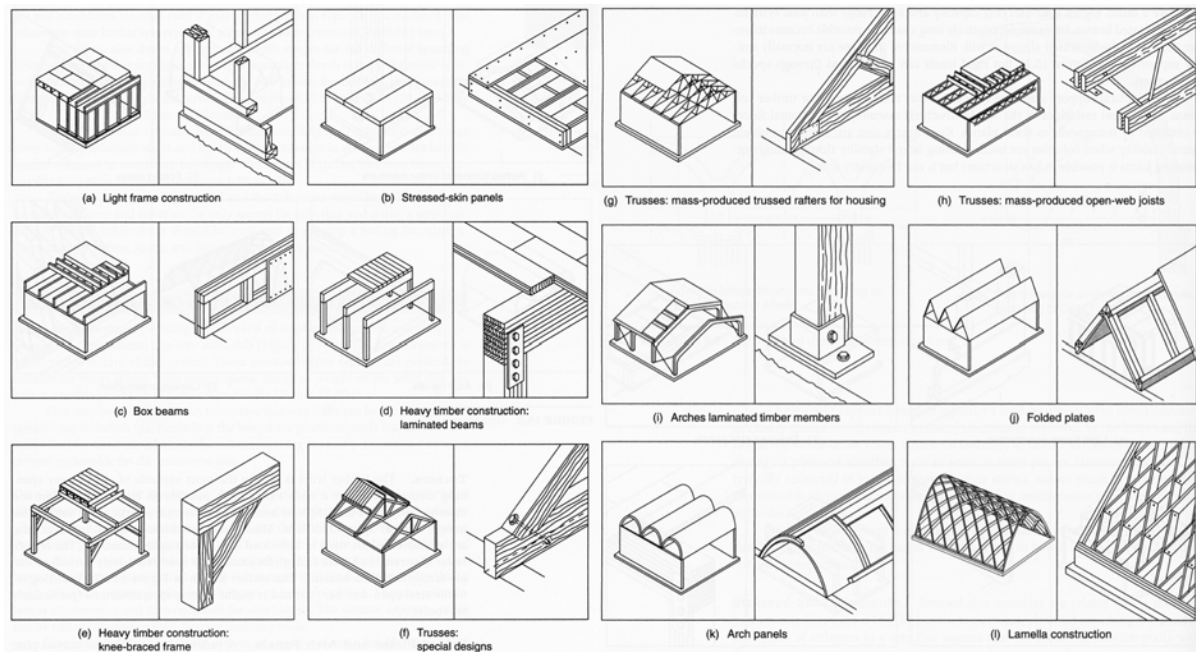


Figure C2.10: Types of Wood Construction. *Structures*, Schodek/Bechthold. Figure 15.2 pp487-88

## REVIEW QUESTIONS

- What are some characteristics of wood that causes it to be a sustainable material?
- What is the most problematic issue of using wood as a structural system?
- Wood is considered a good material (as is steel) for bending strength. Why?
- True or False. Wood has an elastic modulus of  $1/10^{\text{th}}$  that of steel which makes it a stiffer material than steel.
- What are three advantages of glu-lam beams over conventional solid lumber beams?
- What would be more efficient? A stressed skin floor panel construction or one of CLT?

## 2.5 Bamboo

2.5.0 Bamboo might be thought of as wood but in fact it is a grass. It can reach 40m+ in height, 36cm in diameter and weigh 830kg/m<sup>3</sup>. Currently much research and construction experimentation is being pursued with bamboo because of its advantages as a fast-growth natural material with a high strength-to-weight ratio and a stiffness slightly higher than wood. The form of bamboo is unique. It is a tubular structure of segments (internodes) and joints (nodes). It has very little taper although the thickness of the wall of the culm decreases with height. Bamboo is very strong in tension, comparable to wood in compression, and has a high flexural strength. Its slenderness makes it bendable thus enabling curved structures. As with wood, joinery is a major challenge in bamboo construction.

2.5.1 Design with bamboo relies more on precedent than explicit guidelines. The Hong Kong Buildings Department has created a guideline that pertains to bamboo scaffolding (*Guidelines on the Design and Construction of Bamboo Scaffolds*) and contains useful information regarding the support, dimensioning and stability of exterior scaffolding in bamboo. Although information can be found on the material properties of bamboo and composite bamboo products, to be accepted as a sustainable construction system for more than the small detached residence or pavilion, an international standard and code is needed. Especially needed is guidance on the strength and detailing of joints and connections.



2.5.2 Engineered bamboo products provide an improved and more consistent level of performance than natural bamboo poles. Glue laminated bamboo for example, offers the potential of a modularized construction material that is more durable, more resistant to fire, and more predictable in strength and stiffness.



Figure C2.11: Bamboo Structures. Examples.

2.5.3 Bamboo Scaffolding is a cultural tradition of China still practiced in Hong Kong. The lightness of bamboo (about 1/10th that of steel), its high strength and low cost made it an ideal material for temporary construction such as scaffolding. Erection speed is six times that of steel. Installation and dismantlement is performed by specially trained workers who require no special mechanical equipment unlike steel. A bamboo structure is a frame of vertical and horizontal span elements up to 7m in length and connected (tied together) with standard nylon strips 5.5 to 6.0 mm nominal width and a nominal thickness range from 0.85 to 1.0 mm. Continuity of the vertical load bearing columns is achieved by overlap with proper lashing with nylon ties. In addition to scaffolding, temporary bamboo structures are commonly erected in Hong Kong for special ceremonial and religious events (e.g., Hungry Ghost Festival).



Figure C1.12: Bamboo Scaffolding and “Hungry Ghost Temples”. Examples.

## 2.6 Masonry

2.6.0 The stacking of individual units of non-compressible material, both natural and manmade, is referred to as masonry construction. The first masonry units were stone and were quarried with extreme labor. Baked clay bricks and tiles quickly supplanted stone because of the ease of manufacture. Also, in some ancient lands stone was not as accessible. The Romans used stone as a facing material on clay brick structural walls. Since the latter part of the 19c, concrete masonry units have become a dominant load bearing wall construction material.

### 2.6.1 Stone

- The oldest form of masonry construction. The gathering and stacking of *fieldstone* found in the soil and river beds creates rough uneven walls of strength and reasonable durability. The gaps between the rough stones was originally filled with mud. This was later replaced with lime mortar. *Rubble* refers to irregular quarried fragments that have one or more flat faces that can create a more even surface. *Dimension stone* is quarried and cut to rectangular shapes.

- Because of cost, almost all stone now is used for facing rather than structure. Stone can be cut very thin for cladding or paving. For example, granite, a very strong stone, can be cut as thin as about 10mm. Other stone that is less strong must be thicker. Limestone, for example, should be about 75mm minimum thickness. There are also limitations on sheet size. A piece of limestone should be no larger than 1.5m x 5.5m. Recommended maximum for marble is 1.5m x 2.1m.
- The Parthenon is made of Marble. The Great Pyramid and Chartres Cathedral are made of limestone.

### 2.6.2 Clay brick

- Sun-dried clay brick construction is perhaps the oldest known form of building. Clay brick increases dramatically in strength when burnt in a kiln. This is one reason why clay brick is the most fire resistant of all masonry units.
- The size of a brick is small, being dimensioned to fit the human hand. In Hong Kong, the size should comply with the National Annex of BS EN 771-1, Table NA.1 with work size (actual) to be 215 x 102.5 x 65 mm and coordinating size (nominal, includes a 10mm joint) to be 225 x 112.5 x 75 mm, or other dimensions subject to approval by the SO. An alternate size brick of 190mm x 90mm x 90mm is used in the MTR.
- The minimum compressive strength for a building brick is generally about 10-20 MPa. A compressive strength of about 70 MPa is not uncommon. High-strength brick masonry can reach 140MPa. In Hong Kong individual facing bricks should have a minimum compressive strength of 17.2 MPa (ASTM C216). Load bearing brick should be HD Type brick with a minimum compressive strength of 75 MPa (Class B) and 125 MPa (Class A). See BS EN 771-1 Table NA.6.
- The pattern of brick layering addresses structural as well as visual preferences and has developed over centuries. A wall with a single width of brick (a *wythe*, also referred to as a *leaf*) with bricks laid end to end (*stretchers*) and each row shifted a half-brick in length, is called a *running bond* and is the most common type. Other types of walls that are double thickness (2 wythes) or more, employ *headers* which are bricks turned so that their ends are facing outward. There are names for various patterns such as English Bond, Common Bond and Flemish Bond.
- A mortar joint separates each brick and bonds it to the adjacent bricks. The thickness and character of the joint can vary for different visual effects. A standard mortar joint is 10mm. After setting (+1 hour), the joint is *tooled* to provide a neater and better finish. Particular types of joints include weathered, concave, vee, flush, raked, stripped (a deeper raked joint) and struck (opposite of a weathered joint).

*Frank Lloyd Wright consistently preferred a stripped joint for the horizontal and a very thin flush joint for the vertical. This had the visual effect of accentuating the horizontal lines of the brick surface. He also used a much flatter Roman brick with a height of 41mm that increased the number of courses and hence, also the number*

- Openings in brick walls require structural support over the span of the opening. This is normally done with a beam referred to as a *lintel*. Lintels can be made of reinforced concrete, reinforced brick (cement with rebar inside the thickness of the brick wall) or a steel angle (shelf angle) for support. Openings can also be made using the brick as a structural arch. Stylistically, different types of arches may be employed: semi-circular (Roman), segmental, Tudor, Gothic, etc. An arch with a horizontal *intrados* (lower side) is called a *flat* or *jack* arch.

*Louis Kahn preferred the jack arch famously using it on all the openings of the exterior of the Exeter Library. He originally planned to use Roman arches throughout but made the change as a result of cost overruns.*

- Brick cavity wall construction typically uses a gap of 50mm (an airspace between two single wythe walls or between a single wythe brick facing wall and a concrete masonry unit backup wall called a composite cavity wall). The space can be filled with *grout* and vertical rebars at fixed intervals to produce a reinforced masonry wall.

### 2.6.3 Concrete block

- Concrete block units (CMU) are molded with concrete and made in a variety of forms that fit different uses. The first successful block producing machine was invented in 1900 (Harmon Palmer).

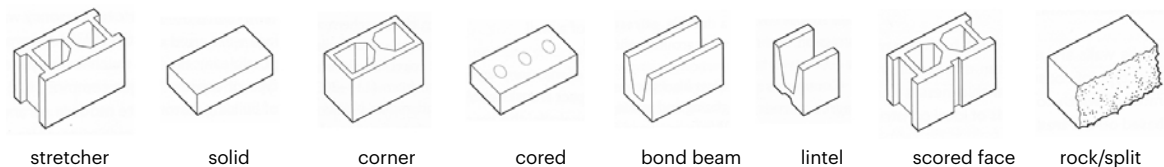


Figure C2.13: Standard Block Types.

- The mean compressive strength of CMU (concrete hollow block) shall be no less than 5.0 MPa of the gross area (*General Specifications for Building* 2022)
- The standard size of a CMU varies depending on the country of origin.

	length	width	Height
United Kingdom	17.22in   440mm	3.94in   100mm	8.66in   220mm
New Zealand	15.35in   390mm	7.48in   190mm	7.48in   190mm
USA	15 5/8in   397mm	7 5/8in   194mm	7 5/8in   194mm

The USA has a modular coordination between CMU block sizes and standard clay bricks. A typical brick dimension is: 7 5/8in x 3 1/2in x 2 1/4in. This means that 2 bricks with 2 standard 3/8in mortar joint will equal the length of a CMU with a 3/8in joint (15 5/8in + 3/8in joint). Also three brick courses (2 1/4in x 3 + 3/8in x 3) is equal to 7 7/8in or about 8in, the height of a standard CMU with a single 3/8in joint.

Figure C2.14: Comparison of standard block dimensions.

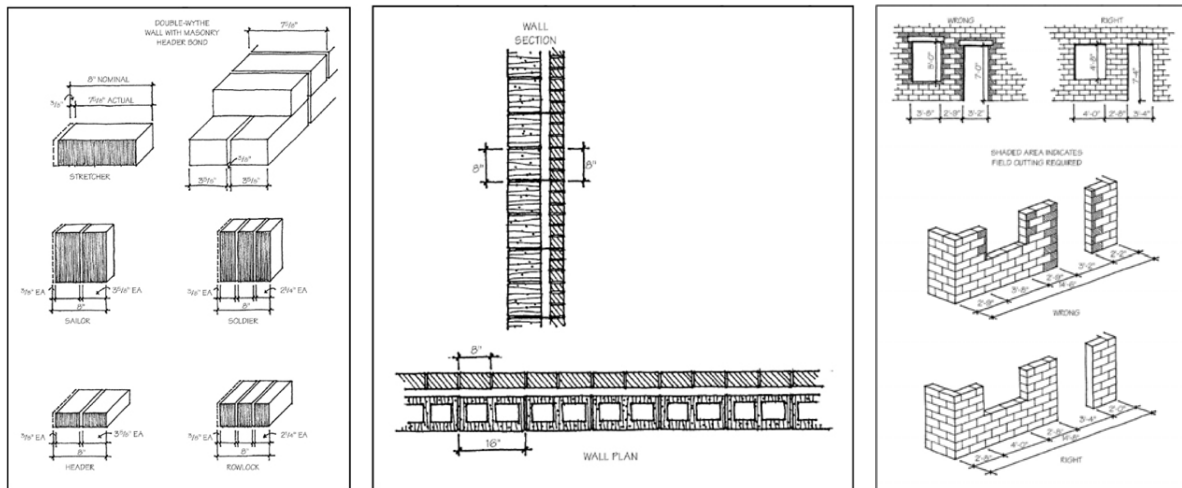


Figure C2.15: Modular dimensioning. *Masonry and Concrete*. Christine Beall

- Vertical movement joints. The expansion and contraction of a masonry wall requires placement of vertical movement joints at set maximum distances. For brick and natural stone masonry the maximum distance is 12m. For concrete CMU it is 6m.



Figure C2.16: Masonry Structures. Examples.

## REVIEW QUESTIONS

- What makes a concrete block such a versatile building material?
- Explain what is meant by “modular brick construction”?
- What is the purpose of mortar in block and brick construction?
- Why is there a 50mm gap in a cavity wall? What is its role?



material	density	yield strength	ultimate strength (tensile/compressive)	Young's modulus E	Thermal expansion	Melting/burning point (loss of strength)
	kg/m <sup>3</sup>	N/mm <sup>2</sup> or MPa	N/mm <sup>2</sup> or MPa	N/mm <sup>2</sup> or MPa	Δ / °C	°C
Carbon fiber (Intermediate Modulus)	1750		4000 - 7000	250,000 - 350,000	1.5 x 10 <sup>-6</sup>	(300)
Steel (Grade 43 or S275)	7850	275	430	205,000	12 x 10 <sup>-6</sup>	1480 (600)
Steel (Grade 50 or S355)	7850	355	450	205,000	12 x 10 <sup>-6</sup>	1480 (600)
Steel reinforcement (F250)	7850	250 - 550	410	200,000	12 x 10 <sup>-6</sup>	1480 (600)
Steel prestress wire (Grade 270)	7850	1700	1920	165,000	12 x 10 <sup>-6</sup>	1480 (600)
Titanium Alloy Ti64	4500	1100	1170	116,000	9 x 10 <sup>-6</sup>	1668 (430)
Aluminium (6061 T6 Structural)	2700	240	260	70,000	20 x 10 <sup>-6</sup>	660 (250)
Glass (Tempered)	2650		120 - 200 / 1000	70,000	9 x 10 <sup>-6</sup>	1600
Concrete (C20)	2350		20	18,700	10 x 10 <sup>-6</sup>	1550 - 1700 (600)
Concrete (C60)	2350		60	30,000	10 x 10 <sup>-6</sup>	1550 - 1700 (800)
Concrete (lightweight)	1600 - 2000		5 - 60	5,000 - 25,000	9 x 10 <sup>-6</sup>	
Stone (granite)	2640 - 2720		130 - 310	54,000	8.5 x 10 <sup>-6</sup>	
Stone (limestone)	2080 - 2720		18 - 230	45,000	7.9 x 10 <sup>-6</sup>	
Stone (marble)	2640 - 2720		52 - 190	50,000 - 65,000	13.1 x 10 <sup>-6</sup>	
Stone (sandstone)	2080 - 2640		28 - 240	20,000	11.6 x 10 <sup>-6</sup>	
Concrete masonry	1200 - 2320		12 - 41	30,000 - 50,000	9.4 x 10 <sup>-6</sup>	1550 - 1700
Clay Brick	1600 - 2240		14 - 140	3,500 - 34,000	5 x 10 <sup>-6</sup>	1540
Bamboo (Betung)	830		220 - 320 / 44	19,500 - 26,000		204 (50 - 100)
Wood (soft: fir <sup>1</sup> , spruce, larch)	510		130 / 48	10,000 - 13,000	3.7 x 10 <sup>-6</sup>	374 - 500
Wood (medium: kempas)	770 - 1120		120 / 66	18,600	9.7 x 10 <sup>-6</sup>	374 - 500
Wood (hard: meranti, merbau, teak <sup>2</sup> )	655		118 / 41 - 65	12,000	7 x 10 <sup>-6</sup>	374 - 500
Polyvinyl chloride (PVC)	1300	31 - 60	66	3,400	54 x 10 <sup>-6</sup>	100 - 260

Values listed in this table are for reference only. Depending on the quality and processing of the material, the values may vary significantly from those listed. Values for wood are compression parallel to the grain.

Figure C2.17: Structural Materials: Properties and Values.

## SELECTED REFERENCE

- 1) *Fundamentals of Building Construction*, Edward Allen and Joseph Iano, 2009 5<sup>th</sup> Ed., John Wiley & Sons.
- 2) *Code of Practice for Structural Use of Concrete*, 2013 (2020 edition). Buildings Department of Hong Kong Government.
- 3) *Code of Practice for Precast Construction*, 2016. Buildings Department of Hong Kong Government.
- 4) *Code of Practice for Structural Use of Steel*, 2011 (2021 edition). Buildings Department of Hong Kong Government.
- 5) *Code of Practice for Structural Use of Glass*, 2018. Buildings Department of Hong Kong Government.
- 6) *General Specification for Building*, (2022 edition). Architectural Services Department, Government of the HKSAR.
- 7) *Design Tech: Building science for architects*, Jason Already and Thomas Leslie, 2007, Elsevier.
- 8) *Building Construction Illustrated*, Francis D.K. Ching and Cassandra Adams, 2001 3<sup>rd</sup> Ed., John Wiley & Sons.



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## Section D Foundations and Site Formation Works

### INTRODUCTION

*Foundations and Site Formation Works* includes geotechnical site investigation, excavation and lateral support systems, and foundation structures.

*Foundations must accomplish three goals.....*

- *the foundation and the subsoil it rests on must be able to resist collapse, that is it must resist the forces that the building structure places on the ground.*
- *the foundation must prevent differential settlement. Uneven settlement of a foundation will cause damage to the building.*
- *the foundation design can not have an adverse effect on adjacent properties or buildings.*

### TOPICS

#### 1.0 Site Investigation

The characteristics of the site that inform foundation design include the above ground topography, existing built structures, presence of water above ground, vegetation and most importantly, the sub-grade condition. Site investigation includes documentary studies, site survey and ground investigation.

#### 1.1 Soil Properties and Characteristics

A basic understanding of the material that constitutes the below ground support of all construction (superstructure, substructure and foundations) is fundamental.

- Soil is any earth material that is particulate. This includes sand, clay, silt, gravel, boulders and peat and other organic matter.
- Resistance to vertical bearing pressure is the primary consideration of soil as a foundation support. In addition, soil must resist horizontal pressure and sliding friction due to lateral loads.
- Dimensional stability: volumetric change due to stress (compaction) or water content (swelling).
- Relative stability: changes in soil due to frost action, seismic shock, vibrations from site work, etc.
- Soil composition: soil particles, air and water. Air and water are considered non-solid and therefore constitute the void space of soil. The amount of void space in soil as a percentage is the *porosity* of the soil. The relative *permeability* of the soil is a measure of the flow of water through the soil.
- Size and gradation of soil particles. Soil particles are grouped according to size. At the lower end of the scale, particles that can pass through a no.200 *sieve* (with the smallest openings) are called *finer*. These particles are known as clay or silt. The next grouping is sand; fine, medium and coarse. If a particle can not pass through a no.4 sieve it is referred to as gravel, either fine (sieve openings between 4.75mm and 19mm) or coarse (between 19mm or 3/4" and 76mm or 3"). Hard particles that are too

large to pass through a sieve of 3" openings are called *cobbles*. Larger stones that cannot pass through a sieve with 12" wide openings are termed boulders.

- Effect of water on soil. Small amounts of water on sand cause the particles to stick together. Larger amounts create a highly viscous fluid. The effect of water on fine grain soils can be dramatic. Fine clays or silts will be rock-like solid when dry and virtual fluids when fully saturated with water.
- Soil shear resistance: *cohesive* (clay and silt) versus *cohesion-less* (sand and gravel). The mixture of cohesive and cohesion-less soils leads to a range of bonding strength. In addition, the presence of water can reduce or fully eliminate cohesion.
- Soil strength varies according to soil type and various conditions such as confinement and water content. Sand, for example has little resistance to compressive stress unless it is confined. The density or degree of compaction of sand will affect also its usable bearing strength. Clay has some resistance to both compressive and tensile stress in an unconfined condition even in the presence of water (due to its cohesiveness).
- Fine-grained soils vary in *consistency* ranging from very soft to hard. The variation in consistency affects the bearing strength of the soil. Another quality affecting strength is the *plasticity* of the soil. Generally the term *silty* indicates a lack of plasticity (crumbly) and the term *clayey* indicates some plasticity (malleability).
- Coarse-grained soils include sand and gravel. Fine-grained soils include silts and clays. Coarse grained soils tend to have higher load bearing capacity, are more stable and less reactive to changes in moisture content.

Type	Classes			Value as a foundation material	Frost Action	Drainage
	Letter	Symbol	Description			
Gravel and gravelly soils	GW		Well-graded gravel, or gravel-sand mixture, little or no fines	Excellent	None	Excellent
	GP		Poorly graded gravel, or gravel-sand mixture, little or no fines	Good	None	Excellent
	GM		Silty gravels, gravel-sand-silt mixtures	Good	Slight	Poor
	GC		Clay-gravels, gravel-clay-sand mixtures	Good	Slight	Poor
Sand and sandy soils	SW		Well-graded sands, or gravelly sands, little or no fines	Good	None	Excellent
	SP		Poorly graded sands, or gravelly sands, little or no fines	Fair	None	Excellent
	SM		Silty sands, sand-silt mixtures	Fair	Slight	Fair
	SC		Clay-sands, sand-clay mixtures	Fair	Medium	Poor
Silts and clays	ML		Inorganic silts, rock flour, silty or clay-fine sands, or clay-silts with slight plasticity	Fair	Very High	Poor
	CL		Inorganic clays of low to medium plasticity, gravelly clays, silty clays, or lean clays	Fair	Medium	Impervious
	OL		Organic silt clays of low plasticity	Poor	High	Impervious
	MH		Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts	Poor	Very High	Poor
	CH		Inorganic clays of high plasticity, fat clays	Very Poor	Medium	Impervious
	OH		Organic clays of medium to high plasticity, organic silts	Very Poor	Medium	Impervious
Highly organic soils	Pt		Peat and other highly organic soils	Not Suitable	Slight	Poor

Figure D1.1: Soil Types Chart (USCS). *Design Tech*, Alread/Leslie. Figure 1.3.1 p69

## 1.2 Subsurface exploration and soils testing

Determining the suitability for construction on a new site and the type of foundation support needed will rely on a subsurface investigation. The soil and water conditions below ground must be known before foundation support can be designed. To find out the load bearing capability of the soil and the level of the water table, either a *trial pit* must be dug (shallow foundations) or *test borings* be made. A test boring will provide detailed knowledge of the types of soils below grade (and hence the soil bearing pressure at various depths) as well as the location of the water table. A portable boring drill (mounted on a truck or tractor) is capable of retrieving soil from as much as 450m below surface.

Soil samples brought up by the boring rig are laboratory tested. The information is presented in a bore log. Depending on the size and character of the site, the geotechnical specialist will determine the position and number of borings required.

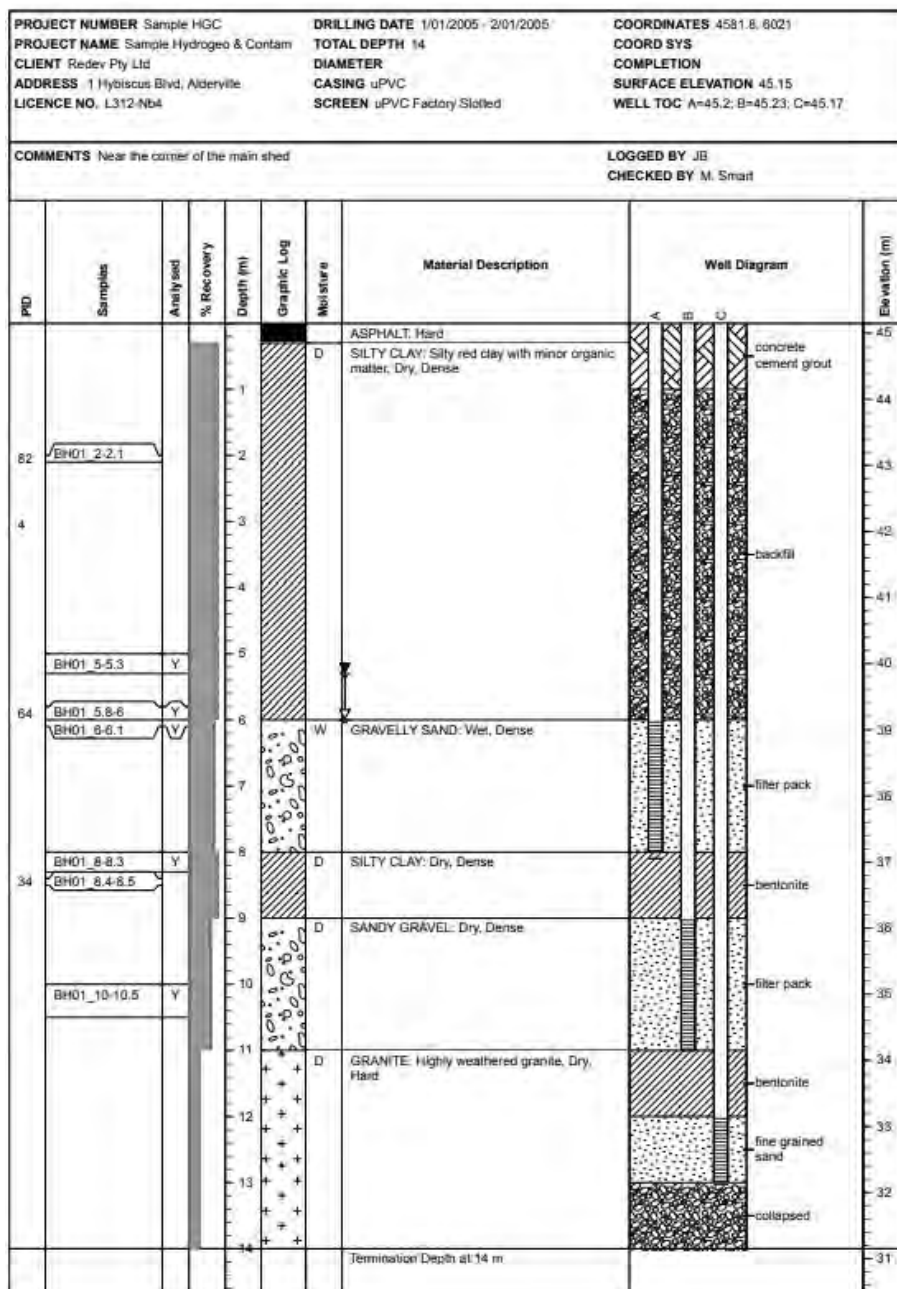


Figure D1.2: Typical Test Bore Log

A trial pit is a type of shallow subsurface exploration typically between 1 and 4 meters deep. It may be hand dug or machine excavated. If there are known or suspected subsurface structures such as utilities, cables, etc. the trial or test pit is dug by hand so as to ensure minimal disturbance or damage. This is common where foundation work may be undertaken in locations close to transportation routes and in densely occupied urban areas where underground infrastructure is prevalent. Larger test pits using machine excavation can range from 1.5 to 6 meters in depth, with 3 meters considered typical. Samples of soil from test pits may be tested for both geotechnical properties and contamination. Since test pits do not normally employ shoring to prevent the collapse of the side walls, it is not advisable to use them if the ground is wet.

The presence and location of groundwater is critical in foundation design and construction. This is especially important where *dewatering* of the site may be required. A simple *standpipe piezometer* is basically a tube with a filtered tip into which subsurface water flows; the height of the column of water in the tube determines the groundwater pressure. A *vibrating wire piezometer* is a device installed in a borehole that measures groundwater level, water pressure and can determine flow patterns.

### 1.3 Allowable Bearing Pressure

The strength of soil (or rock) to support a foundation load is referred to as the *allowable bearing pressure*.

*“Allowable bearing pressure.* The maximum allowable bearing pressure that may be applied at the base of the foundation, taking into account the ultimate bearing capacity of the soil or rock, the magnitude and type of settlement expected and the ability of the structure to accommodate such settlement.” Code of Practice for Foundations 2017

The presumed or approximate value for different types of soil is listed in Table 2.1 of the Code of Practice for Foundations 2017 of the Buildings Department. This chart assigns a value for the allowable bearing pressure based on description. Bearing pressure values range from very strong granite rock (10,000 kPa) to cohesive clay soils at 80-300 kPa. From this information the foundation type and its dimension can be determined.

*Bedrock* (the surface of which is called the *rock head*) is an uninterrupted mass of rock that can support foundation bearing loads without sinking, settling, or shifting. It is inert and does not contract or expand like soil. Bedrock is sometimes exposed above the ground surface or can lie hundreds of meters below the ground level. The COP for Foundations in Table 2.1 describes four categories of rock (granite and volcanic rock) and assigns an allowable bearing pressure to each. The highest is category 1(a) at 10,000 kPa. Following is 1(b) at 7,500 kPa, 1(c) at 5,000 kPa and 1(d) at 3,000 kPa. Category 2 is Meta-Sedimentary rock with a 3000 kPa bearing capacity. Category 3 is intermediate soil (decomposed granite/decomposed volcanic rock). Categories 4 and 5 are non-cohesive and cohesive soils with allowable bearing values ranging between 500 kPa to 80 kPa.

The allowable capacity for soils and rocks may also be estimated by appropriate methods of load testing of the foundation on the site (in situ). Use of the Kentledge block test is a common method. An amount of weight exceeding the estimated load on the foundation being tested is applied and measurement of settlement is used to determine capacity.

### REVIEW QUESTIONS

- Make a list of items that should be included in site investigation. (Consult the Code of Practice on Foundations 2017)
- Is a large stone considered to be “soil”? List all of the particles that can be considered as soil.



## 2.0 Site Preparation

The chief activity involved in site preparation is excavation. The removal of top soil and debris is followed by the excavation of the ground to a depth required for the construction of sub-grade building structure and/or the placement of foundation structure.

### 2.1 Excavation and Lateral Support (ELS)

**Bench excavation.** For sites that are large with adequate land around the footprint of the building, the slope of the excavation formed into a series of steps with alternating flat and sloping portions. The angle of the sloped portion is such that the soil will not fall into the excavated hole. It depends on the cohesiveness of the soil and the angle is called the *angle of repose*.

**Embedded wall.** There are several types of temporary and permanent wall structures to prevent the soil from falling into the excavated site. (*Reference: Deep Excavation Design and Construction. GEO Publication No. 1/2023. See Ch.3 Excavation Support Systems*)

- Channel planking wall. Steel channels (C-shaped sections ranging in size from 150mm x 90mm to 300mm x 100mm) are driven into loose or medium dense soil. Adequate support for depths of less than 4m. No interlock between panels and therefore not water tight.
- Steel sheet metal piling is the most common in use. Steel panels (12m standard length) are folded for stiffness and have edges that interlock and can provide a watertight barrier (Sheet piles are sometimes used for making cofferdams). They are driven, vibrated or pressed into the ground. Structurally they act as a cantilever pile. There are two types: U-shaped (400-500mm in width and 100-200mm in depth) and Z-shaped (700mm x 500mm) profiles that form a continuous wall and have interlocking edges that provide water tightness. Type Z sheet piles have the interlocking edges positioned at the outer and inner faces of the wall, while U-shaped piles locate the edges at the centerline of the wall for more effective shear resistance. The section of the Type Z sheet pile has a greater moment of inertia and sectional modulus than that of Type U, making it more resistant to bending. Hence, Type Z sheet piles are favored for deeper excavation works. The normal depth for sheet piles is 3-15m. For additional strength, sheet pile walls are often stiffened with the addition of *waler* beams. Sheet piles may be reused.

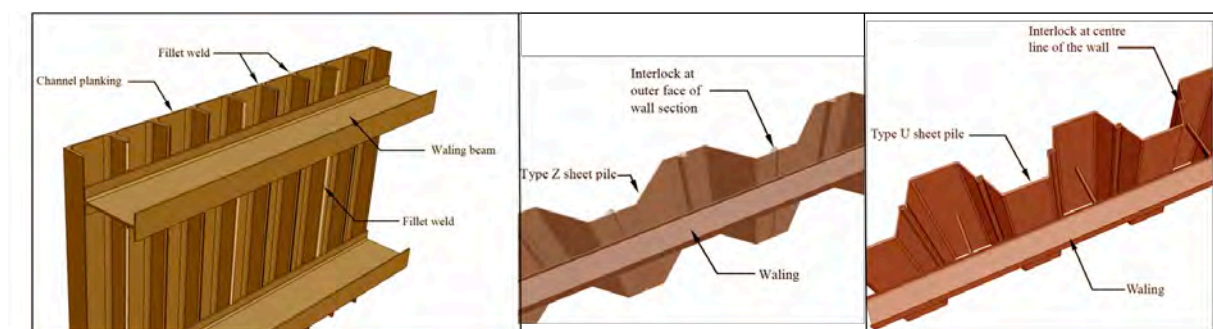


Figure D2.1: Types of metal embedded wall retaining structures

- Soldier pile walls (aka Berlin walls) are typically steel H-piles driven into the ground with some form of *lagging* in between. Wooden planks (often disused railway ties) are traditional but today steel lagging plates are used. Size of H-columns (soldier piles) range from 305mm to 610mm in depth. Soldier piles are typically driven into the



ground (acting as cantilevers) but can be placed in pre-bored holes, backfilled with concrete. Soldier piles may be used in deeper excavations up to approximately 20m.

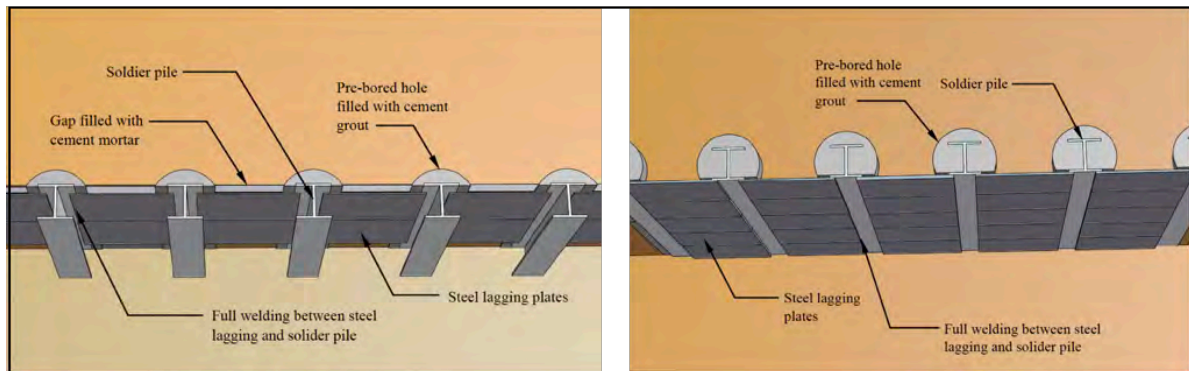


Figure D2.2: Types of soldier pile wall retaining structures

- Pipe pile wall. Similar to a soldier pile wall except that the steel casing tube (219mm to 813mm in outside diameter) used in the boring process remains in position as the vertical cantilevered retaining structure. The gap between equally spaced pipes is filled with steel lagging welded to the pipes. Alternatively, pipes installed with interlocking joints for water tightness are becoming more frequently used on sites that require a groundwater cutoff barrier.

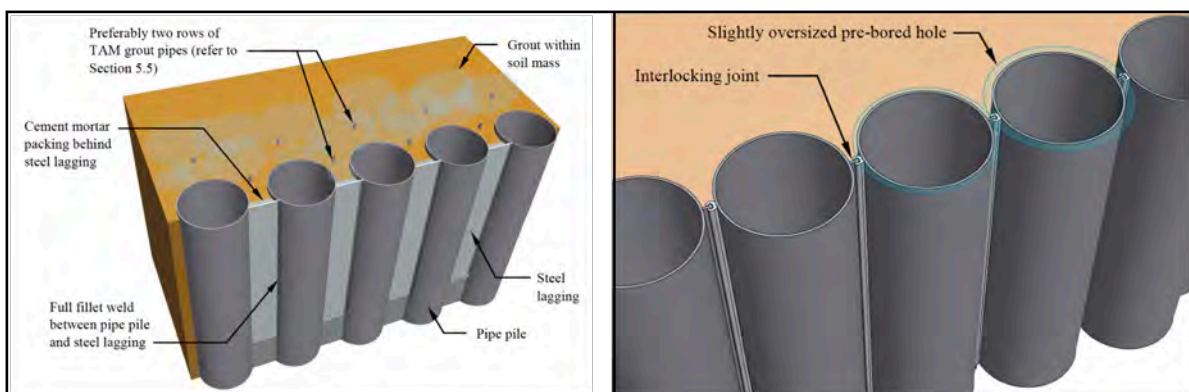


Figure D2.3: Types of pipe pile wall retaining structures

- Bored pile wall. For excavations greater than 15m in depth, a bored pile wall is formed by closely spaced bored concrete piles. Some or all may be reinforced. Small gaps of 100-500mm between the piles may be shot-cretted, grouted or filled with concrete to improve water tightness. A variation is the secant bored pile wall. Concrete piles are cast in two rows, overlapping or intersecting slightly to form a continuous water proof barrier without lagging or filling. A row of “soft” piles using a weaker plain concrete (no reinforcement) are placed first. A second row of “hard” piles on the proposed excavated side are then placed cutting into the weaker piles that have set. The hard piles contain reinforcement and act as the structural support for the wall. Various methods are employed for the boring operation. Some use a steel casing while others may employ a bentonite slurry.

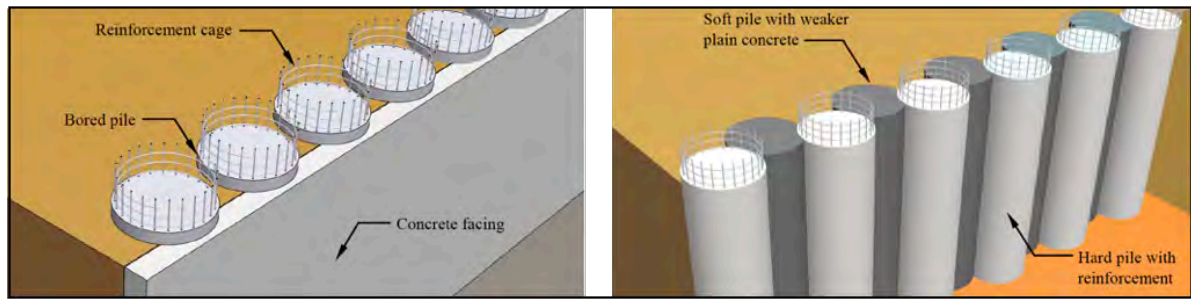


Figure D2.4: Types of bored pile wall retaining structures



Figure D2.5: Bored contiguous pile walls.

- Diaphragm wall. A special type of permanent retaining wall is the *diaphragm or slurry wall*. It is economical for high site retaining walls (walls can extend to about 60m in height). The diaphragm wall is typically a series of aligned, discrete, rectangular reinforced concrete panels of dimensions 0.8m to 1.5m in thickness and 2.8m to 6.4m in length. The process involves digging a narrow trench and filling it with a liquid slurry of *bentonite clay* that prevents the collapse of the trench side walls. Steel reinforcement is then positioned in the slurry after which concrete is pumped in replacing the Bentonite slurry. The slurry is typically captured for future re-use. After the concrete is hardened, excavation of soil on one side begins. The joints between successive panels is made watertight with the incorporation of a steel water stop element. If extending to bedrock, the bottom of the wall is designed to have a shallow rock embedment. Tiebacks are often added to increase the strength to the wall as it becomes exposed on the excavated side

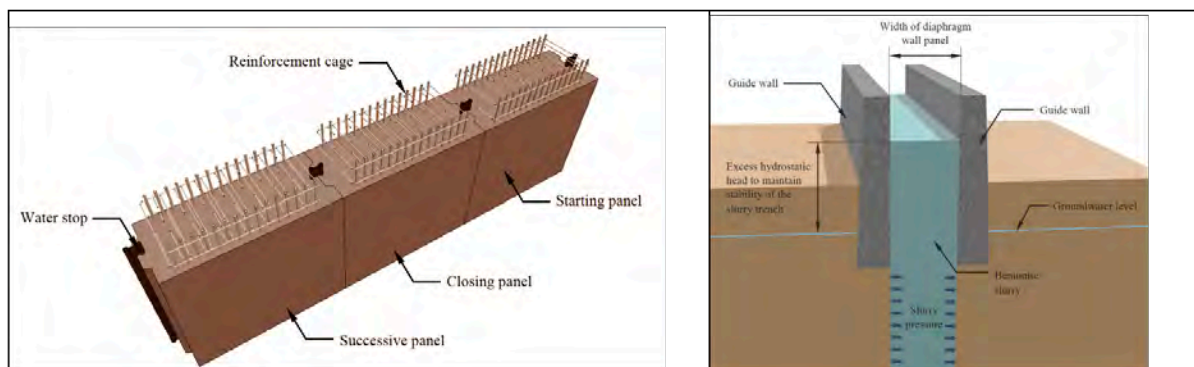


Figure D2.6: Diaphragm wall retaining structures

- Retaining walls fall into four general categories.
  - i) The gravity wall is the oldest type. Generally made of stone or heavy timber, they rely on their weight to resist lateral ground forces. *Gabion* walls, cobble stones contained in wire cages, are a relatively modern example of a gravity retaining wall.
  - ii) The piling wall (H-piles, pipe piles, bored piles) relies on deep penetration into the ground below the level of the excavated side in order to develop a counter moment to the lateral thrust of the ground.
  - iii) The cantilever wall is typically an in-situ poured, reinforced concrete structure that employs a horizontally extended footing to increase its resisting moment to lateral ground forces. Reinforce concrete cantilever walls can be strengthened in different ways to withstand the lateral pressures of soil and water. They can be tapered in profile, buttressed with stiffening elements in the front or back (counterfort) and designed with an extended “toe” (like a bookshelf bookend holder) to prevent overturning. Water accumulation behind the wall is a critical concern and most retaining walls are carefully detailed to provide a means of water drainage. Behind the constructed wall gravel is used in conjunction with drainage tubes to allow the water to collect and flow laterally to the ends of the wall. Also *weep holes* are made in concrete retaining walls to provide openings to allow the water to drain through the wall. Otherwise the accumulation of water will place considerable lateral force on the wall.

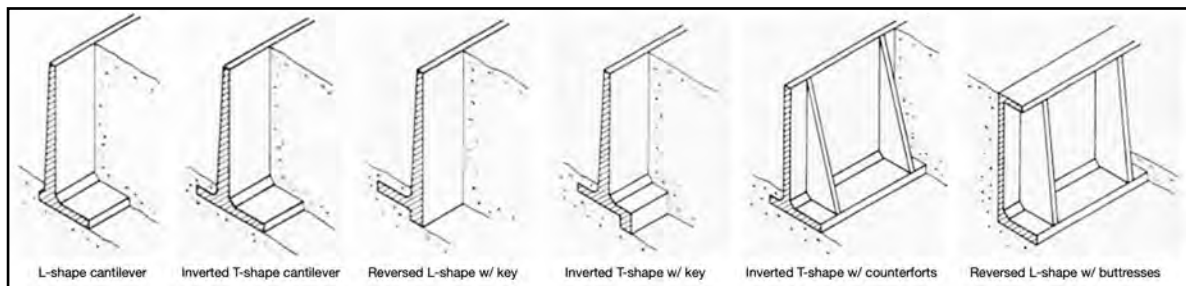


Figure D2.7: Types of r/c cantilever retaining walls

- iv) The anchored wall is a retaining wall structure that relies on the strength of tie-back anchors to resist lateral thrust.

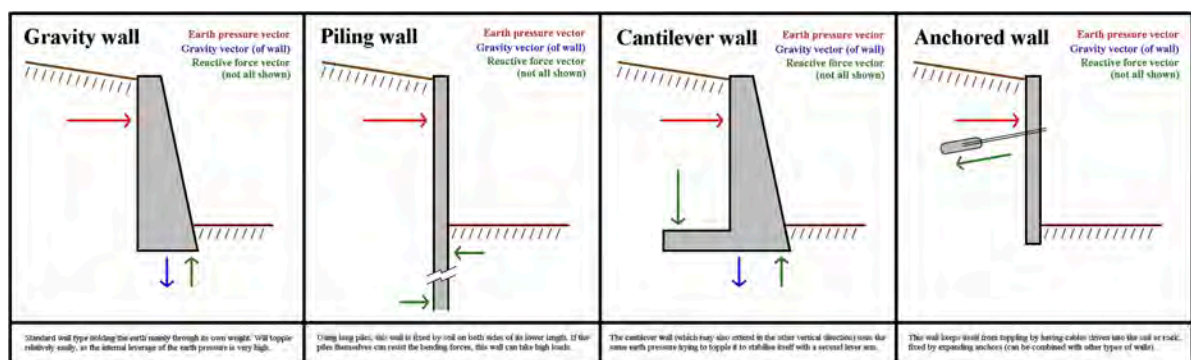


Figure D2.8: General categories of retaining wall structures

*Shoring* is sometimes required to provide additional support to sheeting and other forms of excavation support structures. It can take the form of inclined struts called *rakers* that attach to a horizontal structural member called a *waler* and are anchored at the ground level of the excavation, often by a concrete mass. Sometimes it is more convenient to use



continuous structural members that brace opposite sides of an excavation on the interior. This is called *cross lot bracing*. The disadvantage of both shoring and cross lot bracing is that they interfere with the working space inside the excavated pit.

*Top down construction* is a technique to reduce construction time, especially for buildings that have a deep substructure. In top down construction, the vertical supporting structure, typically in the form of bored caissons that contain the permanent column structure for the lower portion of the building, is introduced into the ground before excavation begins. Excavation then proceeds from the ground level (top) down to the foundations. As the excavation continues, the framing of each sub-floor level is constructed and acts essentially as a form of cross lot bracing. Simultaneously, construction can proceed on the above ground structure, thereby fast tracking the structural construction phase of the project.

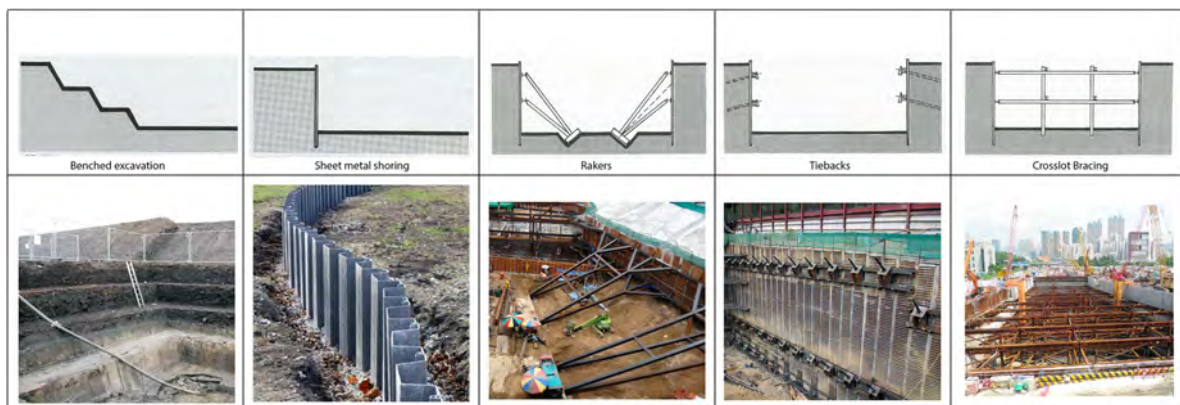


Figure D2.9: General forms of excavation support

## 2.2 Slope Stabilisation

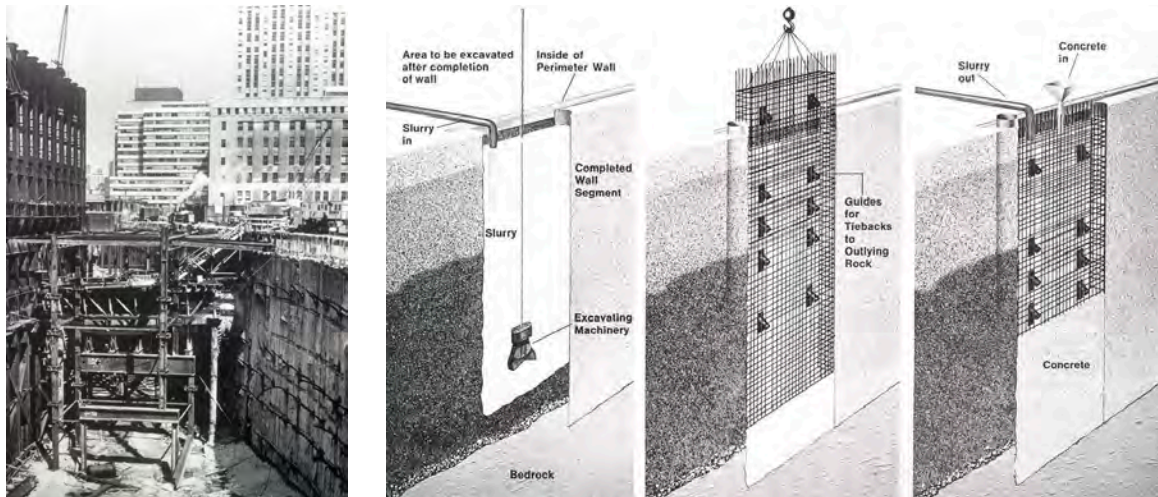
In Hong Kong slope stabilization is a critical concern. Soil on steep slopes may become unstable due to heavy rainfall. Loss of ground cover on slopes may be a contributing factor. Some slopes are manmade as a result of excavation for road building and site preparation. There are several methods in practice to stabilize slopes and prevent minor landslides.

- *Shotcrete*. Perhaps the most ubiquitous method used in Hong Kong on slopes is the spraying of a cement and sand mixture known as shotcrete. The mix is applied to a wire or fibre mesh attached to the slope in a layer of about 50-100mm. Besides improving drainage and preventing the fall of small rocks, the shotcreted surface improves the tensile and shear strength of the slope helping to prevent slides.
- *Textile mats*. Vegetation offers the best option for long-term erosion control on unstable slopes and prevents surface erosion. Geotextile/erosion control mats enable vegetation cover to take hold.
- Other methods of stabilization include *grouting* (to stabilize fissures and cracks in a rock face), *rock bolts and anchors*, *steel rods*, or *cover with concrete grid pavers* (these allow vegetation to grow in the openings) or other heavy masonry units. But these last items only apply to gentle slopes.

## Case Study: Festival Walk, The World Trade Center and slurry wall construction.

The location for the construction of Festival Walk, a large mixed-use commercial center in Kowloon Tong, was a sloping site with an MTR line (Kwun Tong Line) cutting diagonally across. This presented special problems for the design as foundation support would have to straddle the underground railway. To manage the slope, it was decided to use a high retaining wall on the northern side. This wall was constructed as a slurry wall with tiebacks.

The slurry wall is an ingenious process of constructing a retaining wall without the use of shoring. It was used in the late 1960's to create a 23m high retaining wall with tiebacks for the excavation work for the World Trade Center in New York City.



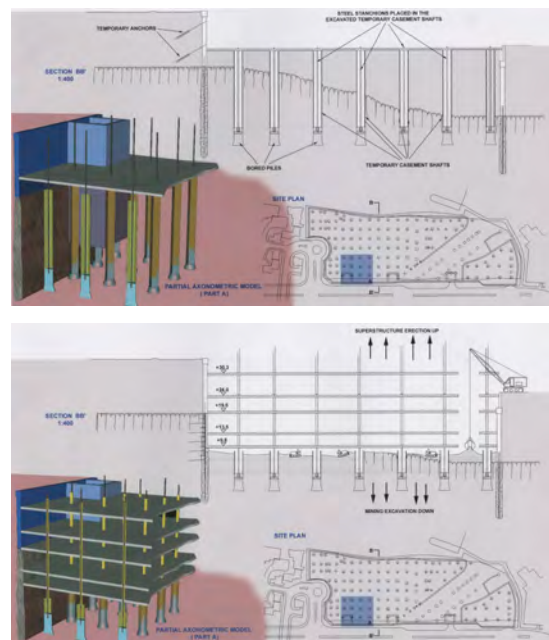
WTC site in New York City ca.1969

Right: Steps in constructing a slurry wall: Using a concrete guiding curb at ground level, a special clam shell bucket digs a deep trench. To avoid the sides from collapsing, the trench is filled with a liquid material called *Bentonite*. A steel reinforcement cage is dropped into the bentonite slurry and then concrete is pumped into trench. As the concrete fills the trench the bentonite slurry is forced out and is retained for further use.



Festival Walk in Kowloon Tong (1993-98).

used at Festival Walk. Note the tieback slurry retaining walls and the bored piles in temporary casement shafts that allow construction on the upper floors while excavating below.



Diagrams illustrating the *top-down construction* approach used at Festival Walk. Note the tieback slurry retaining walls and the bored piles in temporary casement shafts that



## 2.3 Ground Water and Dewatering

The *water table* refers to the upper surface of the zone of saturation, that is the highest point below which the soil or cracks in a rock substructure are filled with water. If excavation goes below the water table on a site, the excavated pit will fill with water. To prevent this there are two procedures. One is to keep the pit dry by using water tight shoring that extends into the ground below the water table and deep enough to penetrate an impermeable layer of soil below grade. The second method is called *dewatering* and involves pumping water out of the ground in order to lower the height of the water table. Pumping is continuous and prevents water from seeping into the excavation.

If a building is relatively light and the substructure of the building lies beneath the water table, a phenomena of *buoyancy* can occur. The building will be pushed upward by the displacement of water under the building. The building will “float” like a boat. To counter this condition friction piles are used. Friction piles resist the upward force of buoyancy on the building. The building can also be made heavier. (See Case Study: Tung Chung MTR)

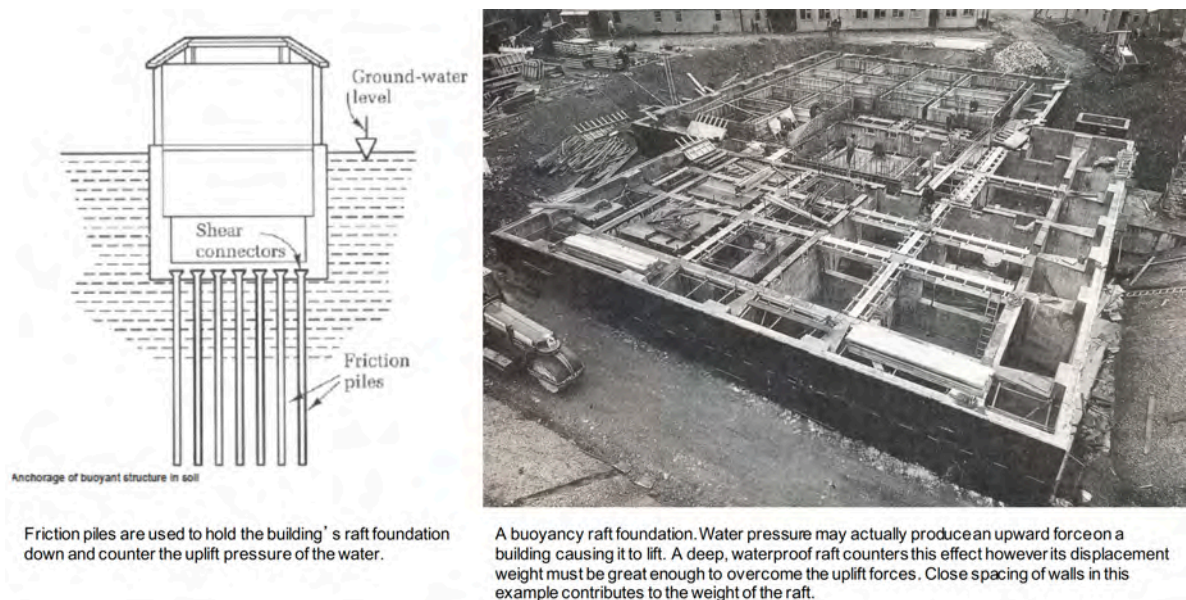


Figure D2.10: Counteracting Buoyancy

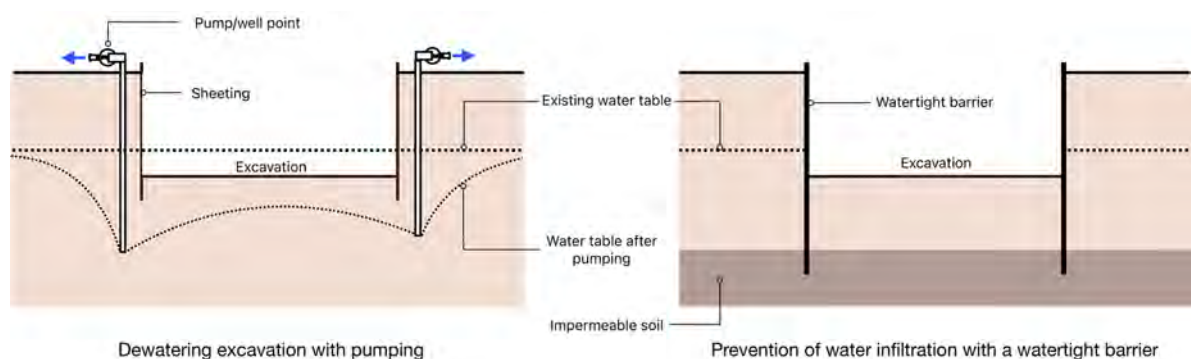
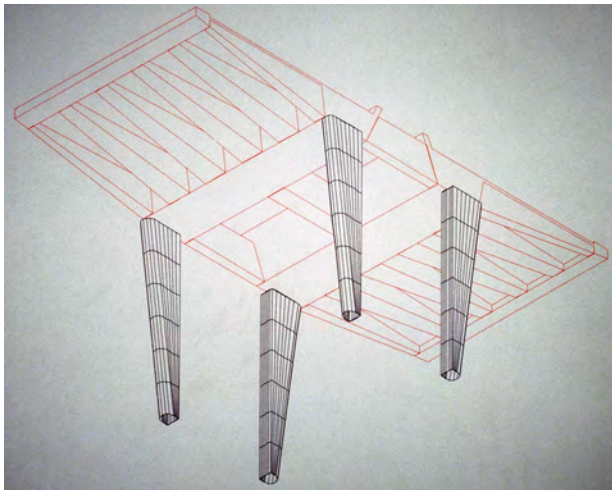
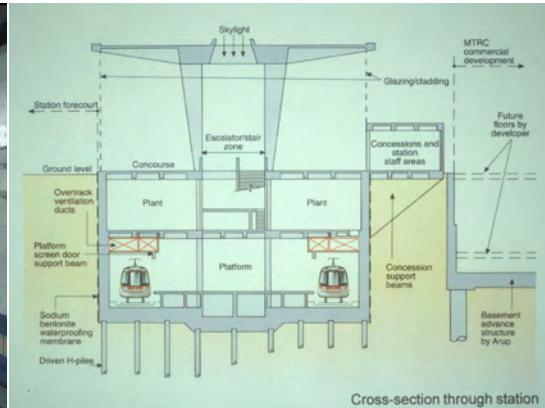


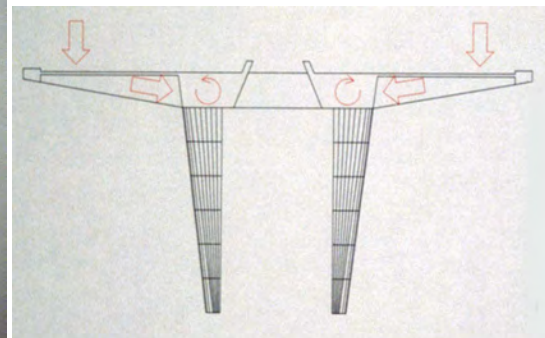
Figure D2.11: Dewatering an excavation

## Case Study: Tung Chung MTR Station

The MTR station in Chung Chung was completed in 1997 and was designed by the office of Rocco Design Ltd. with One Arup HK as consulting engineers. The initial scheme proposed was an elegant steel frame structure envisioned by the designers was light and fully transparent. Early in planning the engineers reported that the building structure as proposed had a critical problem. It was too light! The water table in the island of Tung Chung is fairly high, especially as the site is on reclaimed land. Also the MTR station would be deep with a large substructure to accommodate the platforms and services. On account of this, the building would be buoyant; it would float. The solution was simple. Make the building heavier. The design team switched the structure from steel to an expressive double cantilevered concrete frame, providing the station with glass enclosure on all sides. In addition, the roof profile would include a clerestory skylight.



Clockwise from top left. i) View of interior of station. Section of station with friction piles below train level. Force diagram indication the deep trapezoidal beams resisting torsion. Up view of roof structure.



Above left: Roof structure under construction.

Above right: Specially designed steel forms for the tapered columns.



## REVIEW QUESTIONS

- What is the term “angle of repose” refer to?
- Describe the steps in building a slurry wall.
- Describe the reacting forces to lateral pressure in each of the four types of retaining walls. How does each type resist overturning?
- What are some considerations that would determine whether a benched excavation or temporary shoring is used?
- For a site that has a layer of impermeable soil less than 6m below grade, what would be a simple approach to preventing water from seeping into the excavation? Assume the water table is only a few meters below grade.
- Why was a r/c frame preferable to a steel structure for the Tung Chung MTR Station?

### 3.0 Foundation Structures

Foundation structures support a building and transfer all loads acting on the building safely into the ground. The types of loads that foundations must be able to resist include:

- gravity loads acting on the building or structure (dead, live, snow, etc.)
- lateral loads such as wind, earthquake and hydrostatic pressure (water or soil)
- loads or forces generated by the type of structure such as the thrust of an arch

### 3.1 Types of Foundation Structures

Foundation structures fall into two basic categories: Shallow and Deep

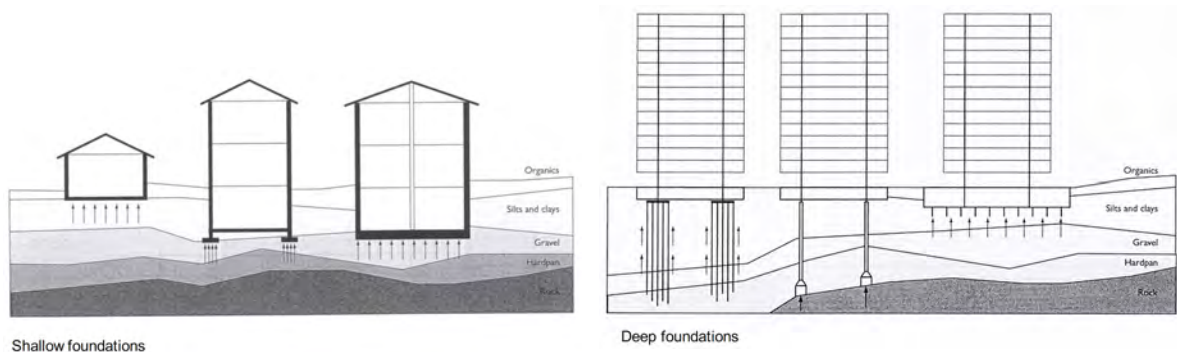


Figure D3.1: Shallow and Deep Foundations. *Design Tech, Already/Leslie* Figure 4.7.5/4.7.7 p325-6

### 3.2 Shallow Foundations

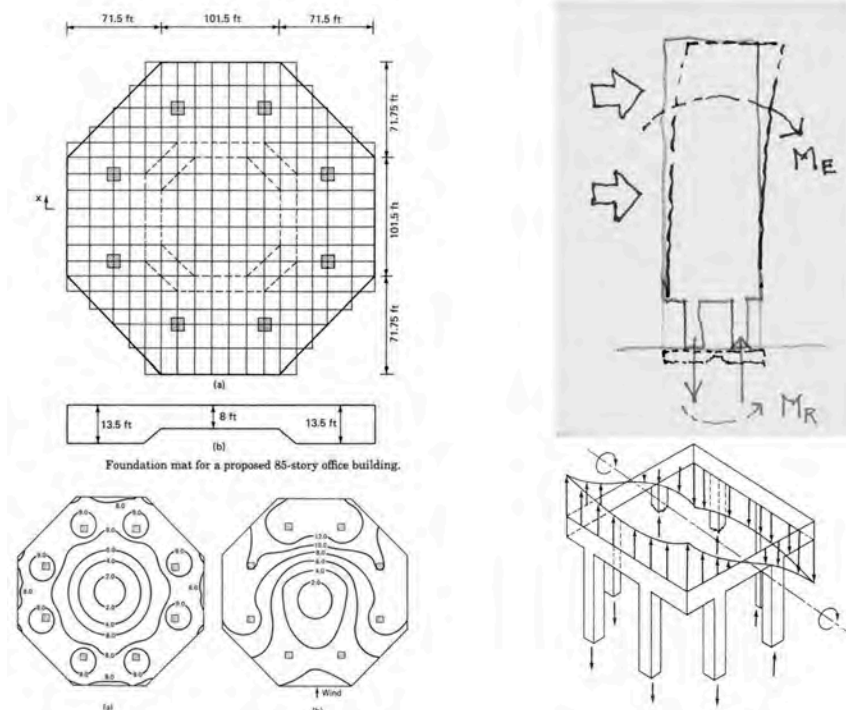
- *slab on grade*: supports light loads directly onto the soil. Underneath walls the slab thickness is usually increased to create a wall strip footing. First the topsoil is removed and a perimeter wall footing with reinforcement and anchor bolts is poured. Inside the footing wall a 100mm layer of crushed stone is compacted over the subsoil to enhance drainage. Above the stone, a layer of insulation (polystyrene) and a moisture barrier such as a sheet of polyethylene plastic is provided. On top of this is placed the steel reinforcement; either deformed bars or welded wire-mesh. Welded wire-mess is normally sufficient for both bending strength and the control of cracking due to thermal expansion. The depth of a slab on grade varies between 100mm and 200mm.

For heavier loading and prevention of cracking due to uneven settlement, a *ribbed slab* with a perimeter edge beam and interior beams in both directions (similar to a waffle slab) may be used. Improved slab strength and elimination of concrete cracks can also be achieved with post-tensioning.



Figure D3.2: Ribbed and PT slab-on-grade

- *raft or mat foundation*: used when the bearing capacity of the soil is low in relation to the weight of the building. Column footings may become so large that it is more economical to merge them into a single continuous footing or raft. Rafts or mats, as they are also called, can be quite thick for tall buildings (>2m). In soft soil the mat is additionally supported by piles. A raft foundation can also be designed as a slab with up-stand beams on top or as a hollow box structure.



Left: Effect of wind on the column loads on the foundation mat. With wind from one direction the forces are much higher on the leeward side of the foundation mat (>12) than on the windward (6). Diagram on right illustrates the forces at the base due to lateral force.

Figure D3.3: Mat foundation for a tall building

- **spread footings** (column/pad or wall/strip): the simplest of all foundations, size or width of the footing is determined by the bearing capacity of the soil. Usually column footings are square unless they are the combined footing type supporting two or more columns. The bearing part of a wall footing extends symmetrically on either side of the wall. The width is commonly twice the thickness of the footing. The bottom of a spread footing should rest on un-excavated soil preferably below the frost line.

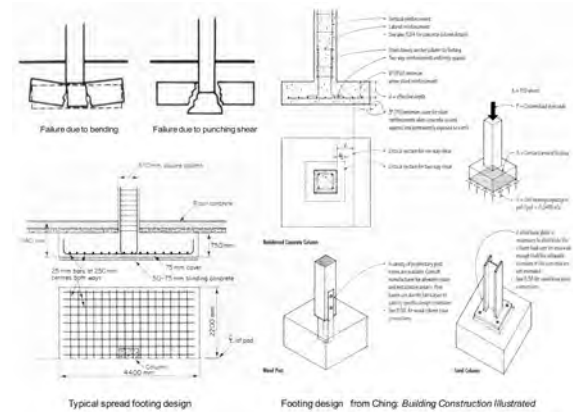


Figure D3.4: Spread Footings. *Building Construction Illustrated*, Francis D.K. Ching.

### 3.3 Deep Foundations

- **piles**: column-like foundations that are driven (hammered) into the ground (displacement) or bored by removing the soil and refilling with concrete (replacement). Small bored piles go up to 750mm in diameter. They can be of various materials: wood timbers, steel H-piles and pre-cast concrete piles of various shapes are a few. There are two basic types of piles, those that are end bearing and those that develop resistance through the friction of the surface of the pile with the surrounding soil. Piles are generally driven in numbers, closely spaced and grouped together by a reinforced concrete pile cap. The pile cap provides a transfer mechanism from a single column to a group of piles. It causes the piles to work together to support the concentrated load from the column. There are three ways to determine pile capacity: 1) allowable pile loads based on the strength of the pile type and the assessment of soil bearing pressure, 2) on-site dynamic load testing (hammer blows), and 3) static on-site load testing (e.g., Kentledge Blocks). For details regarding the allowable load capacity, dimensioning and special requirements of each type, consult the *Code of Practice for Foundations*.

Types of Piles	
5.4.1	Steel H piles or Tubular piles (driven or pre-bored)
5.4.2	Socketed Steel H piles
5.4.3	Precast Reinforced Concrete piles
5.4.4	Precast Pre-stressed spun concrete piles
5.4.5	Driven Cast-in-place Concrete piles ( $\leq 750\text{mm } \phi$ )
5.4.6	Small Diameter Bored piles ( $\leq 750\text{mm } \phi$ )
5.4.7.	Large Diameter Bored piles ( $\geq 750\text{mm } \phi$ ) temporary steel casing used to support the excavation until concreting (or bentonite slurry) end-bearing; socketed with or without bell-out
5.4.8	Mini-piles ( $\leq 450\text{mm } \phi$ ) A number of steel reinforcing bars encased by grout inside a drill hole protected by a steel casing.
5.4.9	Barrettes (slurry wall) uses reinforced guide walls at top to maintain alignment during excavation.
5.4.10	Hand-dug caissons ( $\geq 1500\text{mm } \phi$ , $\leq 3\text{m}$ in depth)
5.4.11	Steel H piles (driven to bedrock)
5.4.12	Steel H Shear piles (for lateral forces only, relatively shallow)



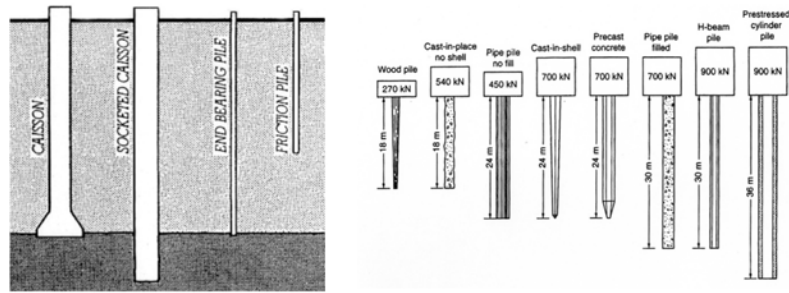


Figure D3.5: Types of Piles. *Fundamentals of Building Construction*, Edward Allen and Joseph Iano. Figure 2.37 p56

- **caissons:** In Hong Kong, caissons refer to hand-dug holes filled with reinforced concrete (now permitted only for holes less than 3m in depth and greater than 1.5m in diameter)
- **large diameter bored piles.** The bottom of a large diameter bored pile rests on a satisfactory bearing stratum, such as rock, dense sands and gravels or firm clay. A temporary cylindrical steel casing is used to prevent the soil from caving in and filling the hole. A belled casing is one whose bottom has been extended outward in a bell shape to provide a wider bearing area. A socketed piling is one whose end penetrates into rock stratum (the hole is drilled) to create a friction-tight end bearing contact with rock. Large diameter bored piles are generally adopted for large column loads such as the main support columns of tall buildings.

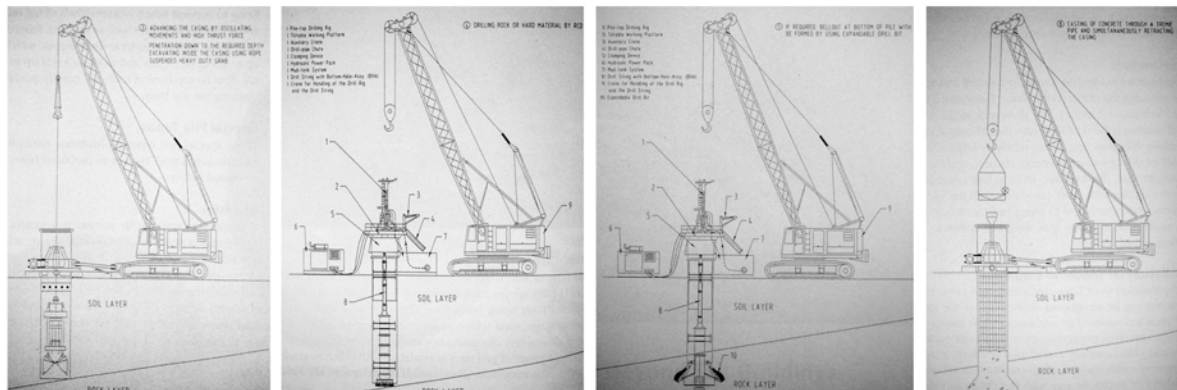


Figure D3.6: Large diameter bored pile sequence

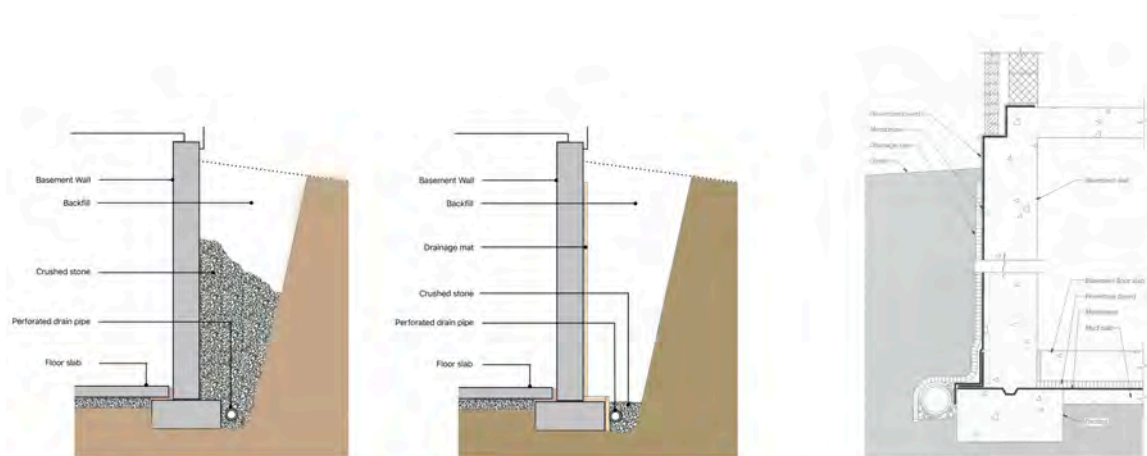


Figure D3.7: Large diameter bored pile work. Tai Wai

- **deep substructures:** sometimes a building substructure is built deep into the earth and requires extensive excavation. A substructure that is several floors deep (e.g., a parking garage) becomes essentially a deep box that may act like a foundation structure distributing the load to the soil below. Often the weight of the excavated soil is equal to or greater than the weight of the complete building, in which case the building foundation can be a raft or mat that is supported by the un-excavated soil below. Since the soil that was excavated was previously supported by the soil below, then it can be assumed that the building of the same or less weight as the excavated soil can be supported if the load is spread equally over the area of excavation. This is known as a floating foundation.

### 3.4 Foundation Materials and Protection

In that foundations tend to be in the soil below grade, they are susceptible to underground forces and moisture. For substructures in particular, the walls and floor must be constructed to prevent water seepage.



Left: Two methods for relieving water pressure on a building substructure by drainage. i) backfill only (difficult to place crushed stone and backfill soil against wall in vertical layers) ii) use of a drainage mat against the exposed wall (easier to place and more economical). Right: Waterproofing a basement wall. A waterproof membrane is placed against the wall extending over basement wall at top and over edge of floor slab. At the bottom of the wall the horizontal membrane under the basement slab extends under the basement wall and joins with the vertical membrane to make a complete enclosure.

Figure D3.8: Drainage and waterproofing on the substructure and foundation. *Fundamentals of Building Construction*, Edward Allen and Joseph Iano, 2009. Figures 2.60 and 2.62, p72-72

### REVIEW QUESTIONS

- Identify the different types of retaining wall structures. Describe important features.
- What is the difference between a pile and a caisson?
- Describe a typical column footing. How might a footing fail structurally?
- Describe some of the detrimental effects on a building of differential settlement.
- What is the difference between a building that is “pushed upward” by water in the ground and a building with a floating foundation?

### SELECTED REFERENCE

- 1) *Fundamentals of Building Construction*, Edward Allen and Joseph Iano, 2009 5<sup>th</sup> Ed., John Wiley & Sons.

- 2) *Code of Practice for Foundations*, Technical Committee COPF, 2017, Buildings Department of Hong Kong Government.
- 3) *Design Tech: Building science for architects*, Jason Already and Thomas Leslie, 2007, Elsevier.
- 4) *Deep Excavation Design and Construction*, GEO Publication No.1/2023, Geotechnical Engineering Office, CEDD, Government of the HK SAR.
- 5) *Structures*, Daniel L. Schodek and Martin Bechthold, 2014 7<sup>th</sup> Ed., Pearson.
- 6) *Building Construction Illustrated*, Francis D.K. Ching, 2001 3<sup>rd</sup> Ed., John Wiley & Sons.
- 7) *Pile Design and Construction Practice*, M.J. Tomlinson, 1994, E & FN Spon.

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## Section E Horizontal Span Structures

### INTRODUCTION

Horizontal Span Structures are structural elements that support loads between points of vertical support. These include beams, trusses, arches, cable structures, plates, membranes and shells. The vertical supporting element may be a column, pier or wall, foundation support, or another span structure.

Horizontal spanning structural systems provide the following:

- support for floors
- support for roofs
- support for the exterior envelope (transfer weight of envelope to columns *and* provide resistance to lateral force)
- support for secondary building elements (stairs, elevated walkways, canopies, etc.)
- support for vertical structure as a *transfer* structure

The basic types of horizontal spanning structures are:

- linear / reticulated: structural elements such as beams, trusses, arches and suspension systems configured such that loads are transferred in a direct and efficient way to the vertical structural supports. Referred to as *framing*. Elements are deeper than they are wide. In the case of arches and suspension systems, depth is considered as either *rise* (arches) or *sag* (cable suspension).
- planar / monolithic and reticulated: slabs, flat plates, steel decks, pre-cast planks, etc. Both one-way and two-way in spanning direction. Elements are wider than they are deep.
- 3D / monolithic and reticulated: vaults, 3D trusses and space frames, membranes, folded plates, domes, etc. Both one-way and two-way in spanning direction. Funicular forms eliminate bending and are the most efficient (use least material).

#### 1.0 Trusses

A truss generally refers to an open web span structure composed of short linear elements connected in a triangulated pattern. The term “trussed” can refer to any solid bodied structure (e.g., columns, 2-way space grid structures, arches, domes, vaults, etc.), whether linear, planar or three dimensional that is formed by a triangulated pattern of linear elements. A typical flat truss is referred to as a *2D or planar truss*. Trusses that are not flat, such as triangular or box trusses are called *3D one-way trusses*. Triangulated structures that have a depth that is very small in relation to their length and width are called *space trusses*.

Trussed structures are composed entirely of straight, axial force (tensile/compressive) members. They belong to a classification of *vector active* structural systems (see *Structure Systems*, H. Engel. 1967).

#### 1.1 Formal Characteristics

- All structural members of a truss are connected together to form triangles. This is the critical feature of the truss: the triangular pattern is geometrically stable and therefore rigid, resisting deformation. This gives trussed structures great stiffness.



- While beams can support either uniformly distributed loads or concentrated loads, trusses are designed to support concentrated loads only at the joints or panel points.
- Individual members are not subject to bending stress and therefore remain straight. A truss as a whole deflects under loading as a result of the elongation and shortening of the axial force members. This distorts the geometry of the truss (although not perceptible) which results in deflection over the span.

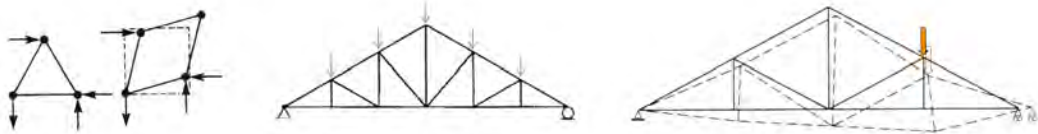


Figure E1.1: Stability of triangle. External loading on joints. Deflection.

- All of the joints of a truss are considered pin-connections or hinged joints. A hinge-type connection allows full rotation of the ends of the members at the joint but restricts translational movement in the x-y-z directions. This means that the axial force member will not be subject to any bending moments but only to axial forces, either compression or tension. In practice, many trusses are designed with fixed joint connections, however the slenderness of truss members is such that bending moments remain small and negligible.



Figure E1.2: Types of connections: fixed joint (left) and hinge joint (right)

- Identification of truss components and the names of the major types of trusses.

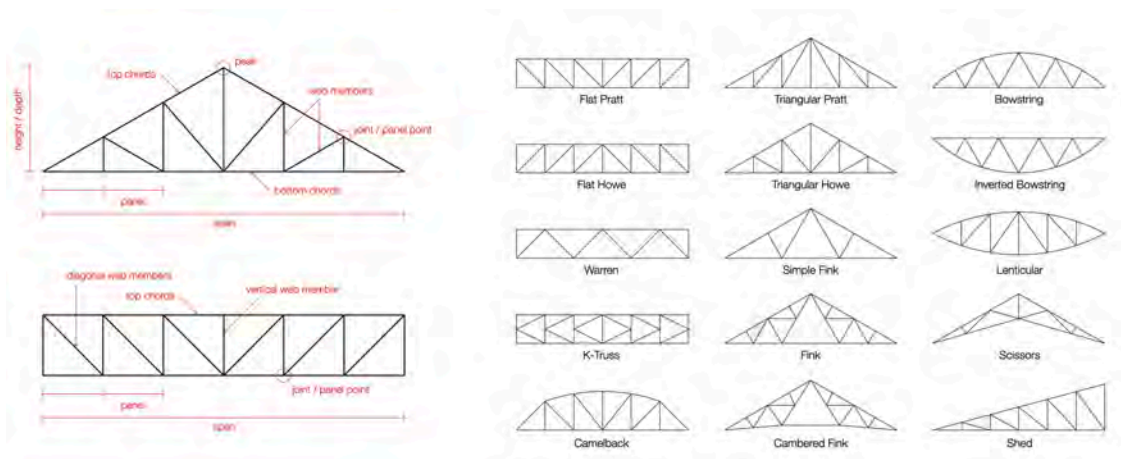


Figure E1.3: Components and standard truss types



## 1.2 Structural Behavior

All span structures must resist bending caused by external applied loads (external applied bending moment  $M_E$ ). The truss develops an internal *moment resisting couple* ( $M_R$ ) with the depth of the truss as the moment arm of the force couple. For positive bending, the force couple will have a compression force at the top and a tension force at the bottom, similar to a beam. An increase in loads or span distance, both of which increase the external applied moment, can be offset by an increase in the depth of the truss, that will increase the internal resisting moment.

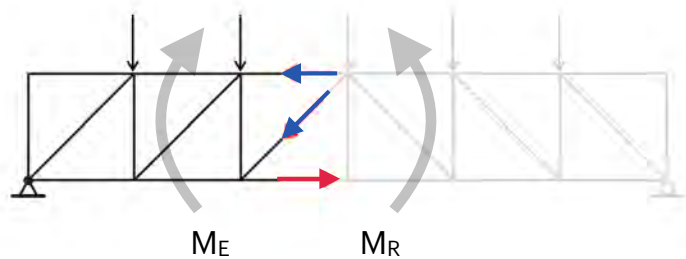


Figure E1.4: Internal member forces and resisting moment

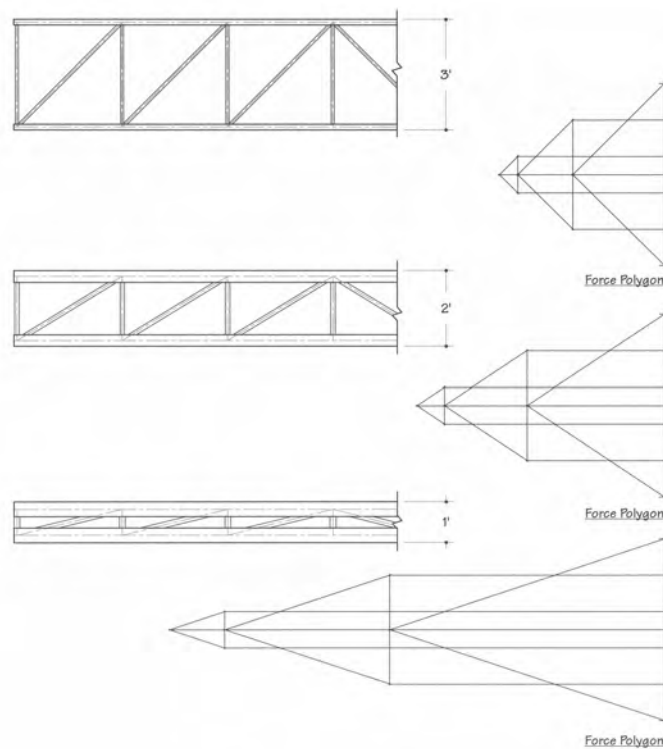


Figure E1.5: Effect of increasing the depth of a truss on member forces.  
*Form and Forces*, Allen/Zalewski. Figure 6.35 p167

An interior web member of a truss may be in tension or compression depending on the configuration of the truss and the loading. The external applied shear force is resisted by the vertical components of these vector forces. Depending on the loading and configuration, some truss members may have no axial force. These are called zero force members. They are necessary for stability and may develop internal force if the loading changes.

### 1.3 Analysis of trusses

To find the forces in a truss of known configuration, dimensions and external loading is a relatively straightforward procedure. Applying the principles of equilibrium, namely the sum of the forces in the x and y directions, and the sum of the moments, the internal axial forces in a truss can be determined. Each of the three methods of truss analysis use the technique of the *free body*, isolating a portion of the truss and solving the equations of equilibrium to obtain the internal member forces required to maintain equilibrium of the truss fragment.

The *method of joints* isolates each joint of the truss in succession. Using the sin and cos of the angles of the truss members with respect to the x-y coordinates, together with the two of the three equations of equilibrium ( $\sum F_x=0$  and  $\sum F_y=0$ ), the unknown member forces can be found. The *method of sections* assumes a portion of the truss as a free body. A cut is made through three members of the truss whose forces are unknown. Using the third equilibrium equation,  $\sum M = 0$  (in addition to  $\sum F_x=0$  and  $\sum F_y=0$ ), these unknown member forces are determined.

The *method of graphic statics* is a third method for obtaining the internal forces in a truss. Essentially a vector representation of the method of joints, the graphic statics diagram presents both the external forces (loads and reactions) and the internal member forces as a single line drawing from which the magnitude of these forces can be measured by scale. At the time when trusses were first being introduced in the 19<sup>th</sup> c as major spanning structures for bridges and long span roofs, the graphic statics method was adopted for designing trusses as the diagram not only revealed the forces of all the members simultaneously, but can be manipulated to improve the structural efficiency of the truss.

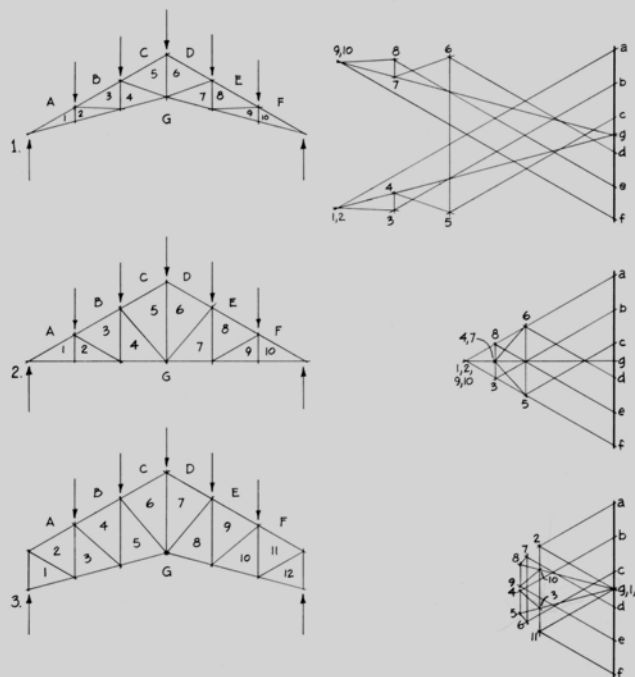


Figure E1.6: Graphic analysis of a truss demonstrating design optimisation.  
*Form and Forces*, Allen/Zalewski. Figure 10.33 p289

Truss analysis today is easily accomplished with the computer using structural analysis software. Once the configuration, dimensions and loadings are input the internal forces and reactions are obtained in graphic form. With the input of member data (material and sectional properties) the deflection shape of the truss is also calculated.

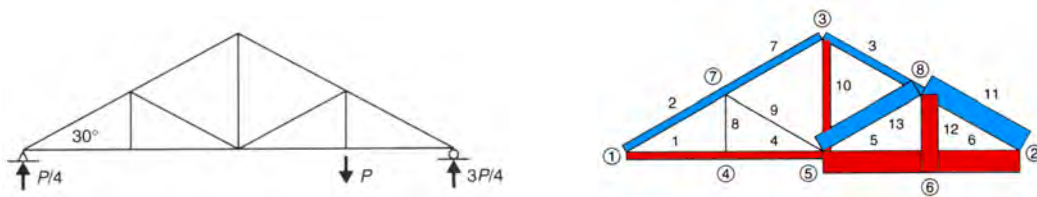
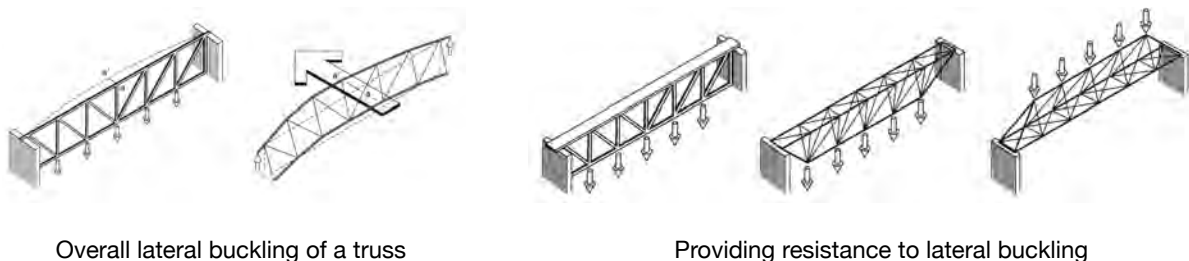


Figure E1.7: Truss analysis with software. *Structures*, Schodek/Bechthold. Figure 4.22 p150

#### 1.4 Stability of trusses

There are two types of stability related to the truss: *overall* and *member*. Overall truss stability refers to the tendency of a truss to buckle laterally. This is due to the upper chord of the truss acting as a long compression member. Under load, the internal compression force in the chord may cause the member to buckle sideways. It cannot buckle in the vertical plane because the interior web members act as bracing members preventing it from bowing upward (or downward).

In most trusses, there will be members framing into the joints of the upper chord and these will provide lateral bracing. However if the truss is free standing, with no members connected to the top chord, then it is at risk to buckle. This situation often occurs during construction when the truss has been erected but the supporting roof or floor members has not yet been attached. Temporary bracing must be used during this critical phase. Several ways to prevent lateral buckling include stiffening the upper chord member or using a three dimension truss. If the truss has a double chord member on the top, this is equivalent to stiffening the upper chord member. If the double chord is on the bottom, the sloping interior chord members create a rigid triangle in section that provides resistance to buckling.



Overall lateral buckling of a truss

Providing resistance to lateral buckling

Figure E1.8: Lateral buckling of a truss. *Structures*, Schodek/Bechthold. Figure 4.34-4.35 p165

In a truss, members with large axial compression force are susceptible to buckling. To prevent buckling, the member must be designed to resist the compressive buckling force, which is much lower than the allowable axial compression force without buckling. Alternatively, critical compression members can be braced. A third strategy is to configure the truss geometry such that compression members are shorter in length. Shortening the unbraced length of a compression member by half increases the allowable buckling load by the inverse square or 4 times.

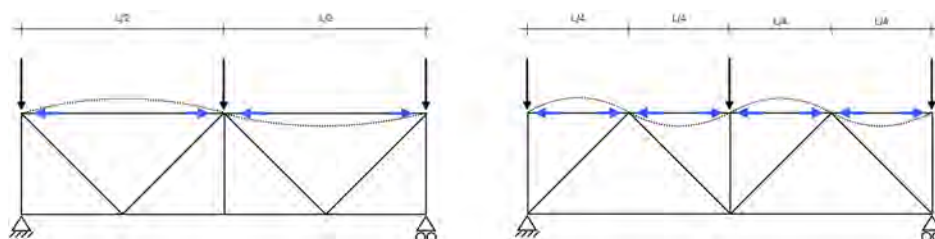
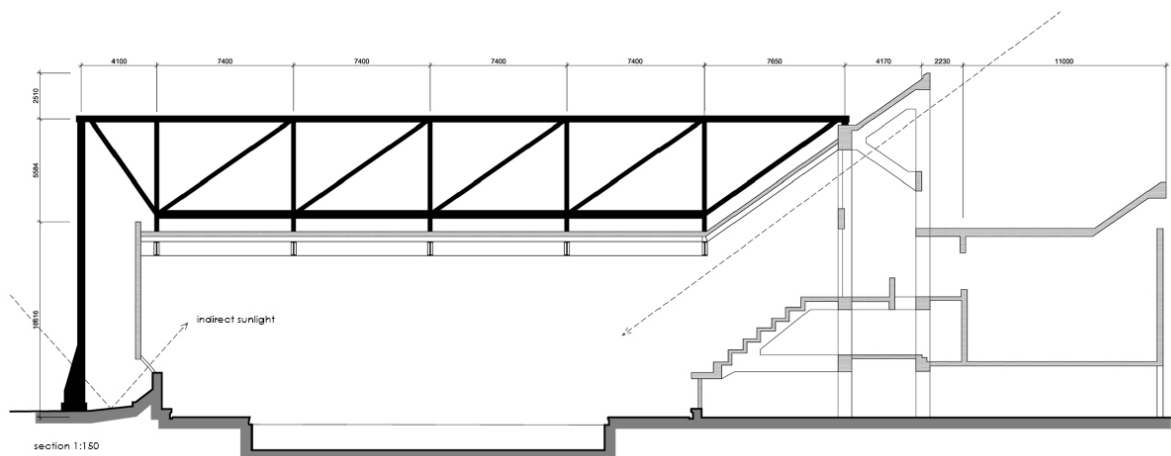


Figure E1.9: Truss member buckling: long compression chords versus short compression chords

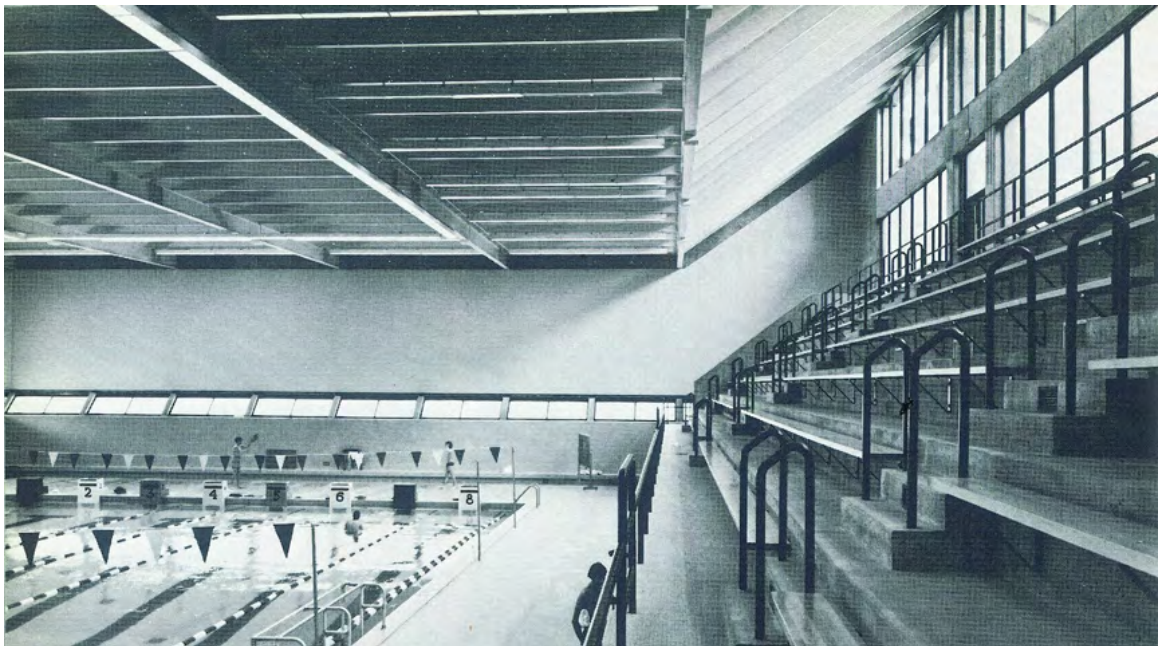
### Case Study: Sports Hall at The Philips Exeter Academy in Exeter, New Hampshire, US

The Philips Exeter Sports Hall was designed by the firm of Kallman, McKinnell and Wood and constructed between 1967 and 1970. It was built at the same time and close to the Exeter Library by Louis Kahn. The Sports Hall consists of indoor ball courts, hockey rinks, squash courts and a swimming pool.

The design of the building uses long-span three-dimensional trusses positioned above the roof, which is supported by steel framing underneath that is connected through the roof membrane to the bottom chord panel points of the trusses. The strategy of placing the deep one-way trusses above the roof reduces the enclosed volume of the building, saving on the cost of vertical cladding and heating/cooling. The trusses also give the windowless sports hall a distinctive visual feature.

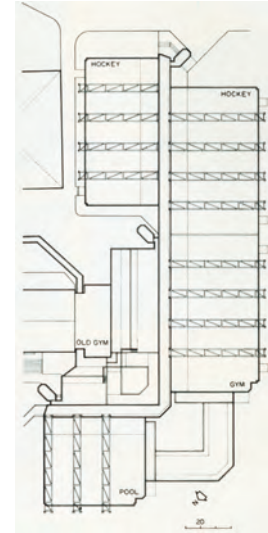
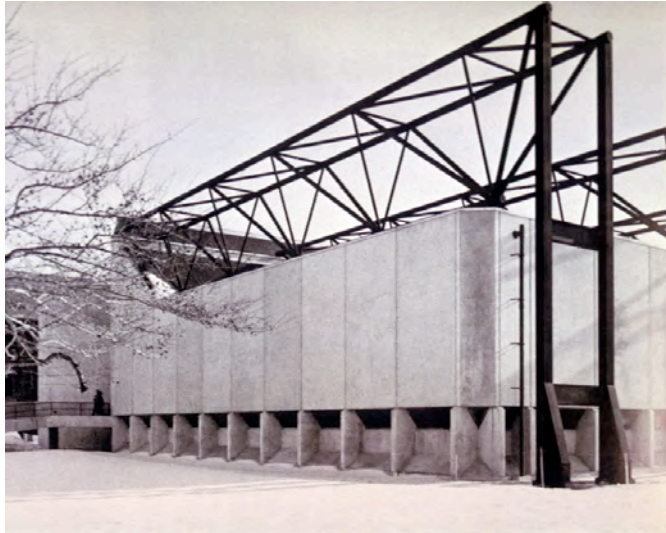
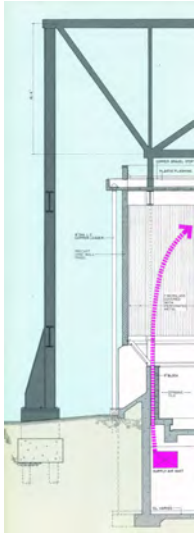


The roof trusses are supported by steel frames of paired H-columns on the outside and a tall, concrete frame at the inside. The frame is glazed above to let light pass through the circulation gallery into the pool. Reflected light off the ground passes into the hall through a continuous strip window angled inward to prevent direct light from entering.



View of the pool showing the exposed steel framing on the underside of the roof. The concrete frame that supports the trusses and frames the circulation gallery also provides the required lateral resistance to the structure.

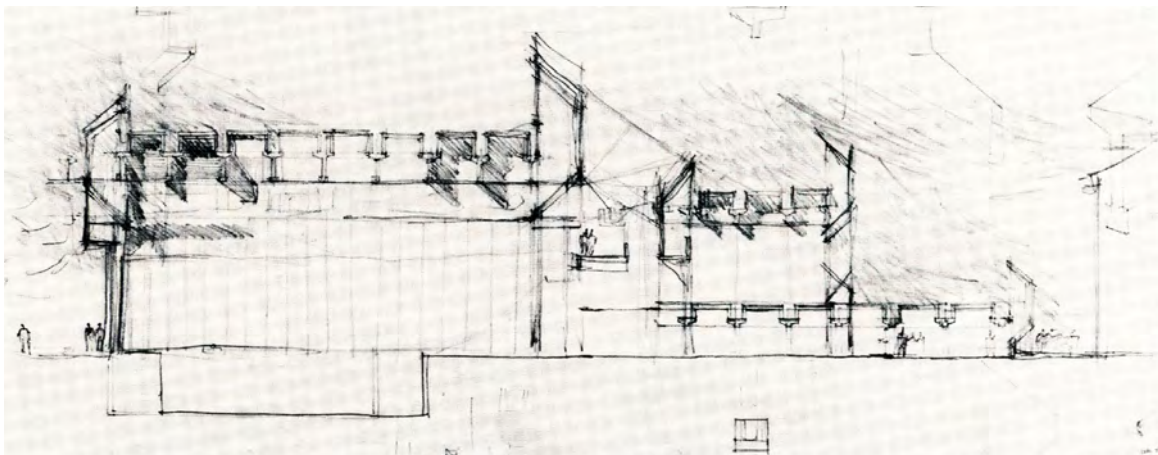
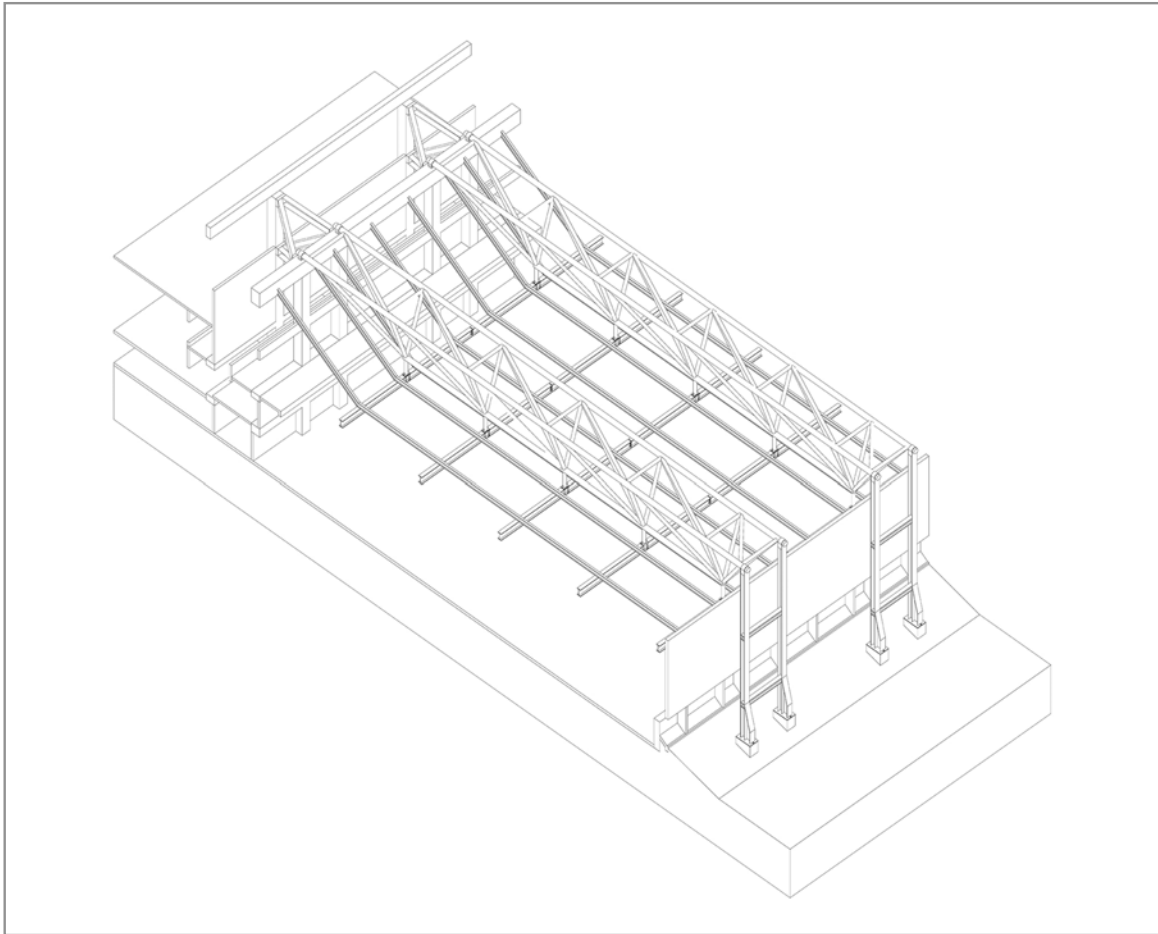




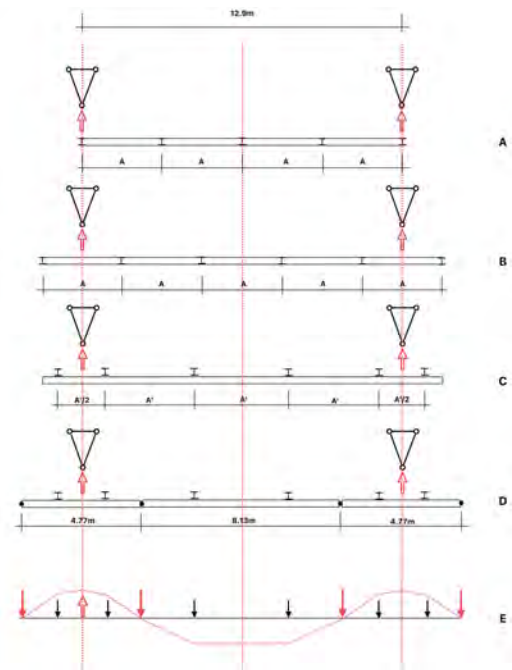
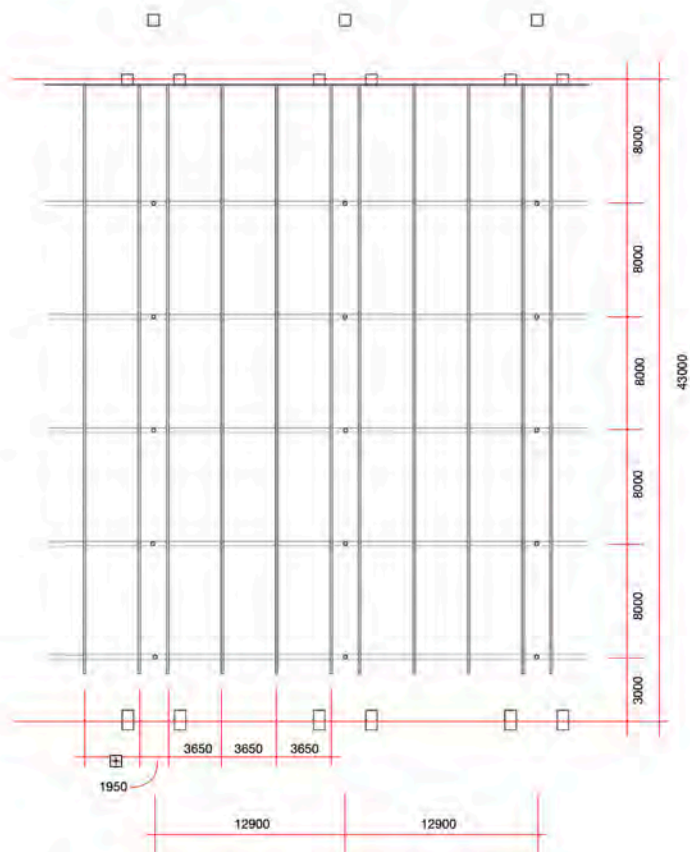
Paired H-columns supporting the trusses outside the building envelope are connected at mid-height and at the height of the horizontal strip window effectively bracing the columns in the weak direction parallel to the face of the building. The flared steel sections at the base act as mini-buttresses providing lateral resistance in the direction perpendicular to the face of the building. The roof plan on the right shows the repetitive roof trusses extending perpendicular to the circulation spline as the layout of the new sports halls wraps around and attaches to the existing gym.



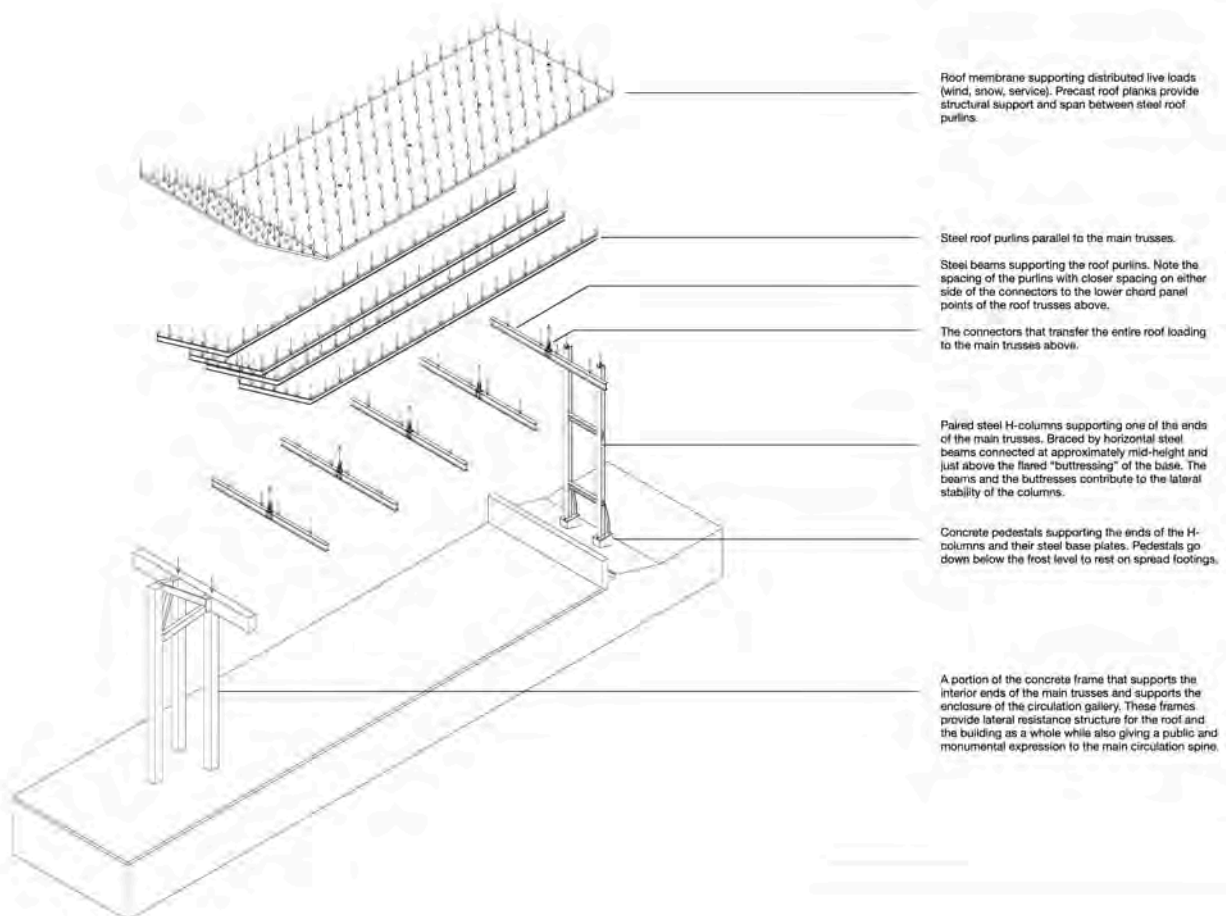




In this early design sketch the building has a more conventional roof framing of deep beams and girders with skylights above. The exterior wall of the building is supporting the roof span structure on the left. On the right, the idea of a light-filled gallery with elevated circulation walks appears but has not coalesced into the powerful three-dimensional concrete spline that anchors the structural support and defines the circulation.



Conceptual transformation of roof framing: A) Typical equal spacing of beams. B) Staggered spacing (positions of beam loads closer to truss supports). C) Beams positioned top of girder and spacing changed to move loads closer to truss supports. D) Hinges inserted in a "Gerber Beam" arrangement (cantilevers with simple beam between). E) Moment diagram (better distribution of moments)



## 1.5 Material and Detailing

The earliest trusses were constructed of heavy timber. In the early 19 c the use of wrought and cast iron was introduced in bridges and some building structures. Later in the 19 c steel was adopted and reinforced concrete and aluminum in the 20 c. Concrete has been used for trussed span structures but technically these should be considered frames as hinged joints are difficult to achieve in concrete construction. The ubiquitous Vierendeel truss that has no diagonal members is also a frame.

Today the selection of material for a truss is largely driven by aesthetics although economy usually favors steel because of the ease of making pinned connections, the slenderness of the sections and the long term maintenance, especially for structures that are exposed.

Truss connections involve careful detailing to achieve the desired joint behavior and the safe transfer of force. Actual “pins” made of steel are sometimes used to achieve full member rotation. However most truss connectors rely on the elastic flexibility of the connection material to provide a slight degree of rotational freedom. Flat steel gusset connection plates for wood trusses are an example.

## 1.6 Advantages and Disadvantages of Trusses

<i>advantages</i>	<i>disadvantages</i>
Lightweight and efficient.	Fabrication of individual members and connections costly.
Assemblage of pieces: can be disassembled and relocated. Also easier to construct.	Individual compression members must be designed for buckling.
Analysis of member forces easy because structure is statically determinate.	Not as stiff as a beam due to number of joints, therefore deflection can be an issue.
Flexibility: any shape is possible therefore profile can easily be shaped as a funicular form.	Susceptible to overall lateral buckling. Bracing is an important concern.
Open web of truss can accommodate services.	Trusses have greater depth than beams.
Trusses carry concentrated loads easily.	Trusses not designed to carry uniform loads.

## REVIEW QUESTIONS

- Can a truss support a uniform distributed load?
- If a triangular panel in a truss is subdivided by the introduction of another member without changing the location of loads on the truss, what would be the force in the new member? Tension? Compression? Other?
- Identify 5 advantages of trusses.
- What are two instability issues associated with a truss? Do they also exist in beams?
- Why do trusses use crossed steel cables in some panels?
- What type of connections are employed in trusses? Why?

## 2.0 Arches

An arch is a linear spanning form, generally curved, with a high point that determines the *rise* of the arch, and descending in both directions to *springing* points at the supports. It carries loads primarily in compression and develops maximum compressive force or *thrust*, at its endpoints.

### 2.1 Formal Characteristics

Arches traditionally were composed of “prefabricated” pieces of stone (the French name is *vousoir*) placed side by side and bearing on each other. The stones formed a specific profile based on the shape of each *vousoir*. The Romans, for example, preferred arches in the form of a semi-circle, so the stone *vousoirs* were shaped identically (which simplified fabrication and construction) and cut in such a way that when assembled, the stones followed a radial curve. Historically different forms of arches have evolved in different cultures. Some are structurally conceived (e.g., segmental) while others are more decorative (e.g., trefoil).

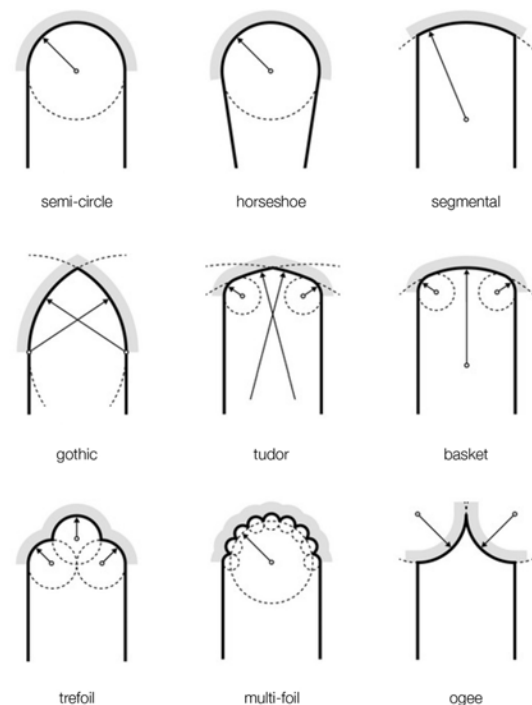
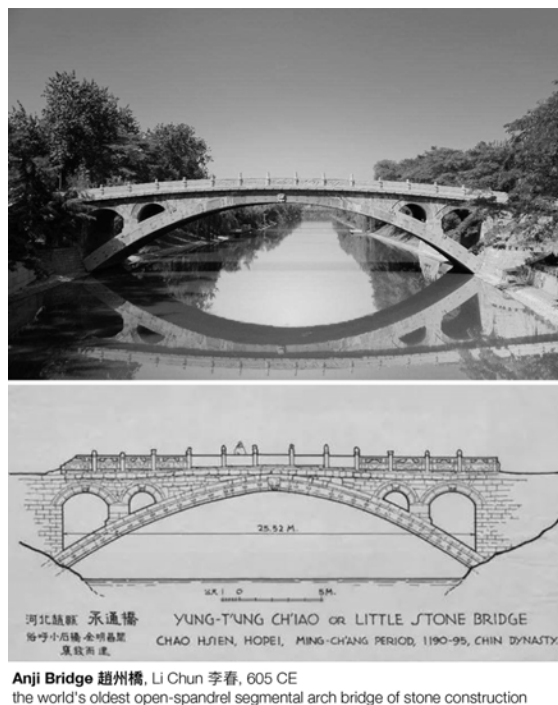


Figure E2.1: Arch forms

Contemporary arches tend to be *monolithic*, that is, continuous but not necessarily of one piece. The size of modern steel arches requires that they be assembled on site from a few segments. Steel arches can be easily welded or bolted together. Concrete arches may be monolithic if continuously poured on site. Or they can be formed of individual precast units and then post-tensioned. Sustainable wood in the form of glu-lam (glued laminated) is commonly used for arches as the laminated wood can be easily fabricated into any profile and the connections are relatively simple.



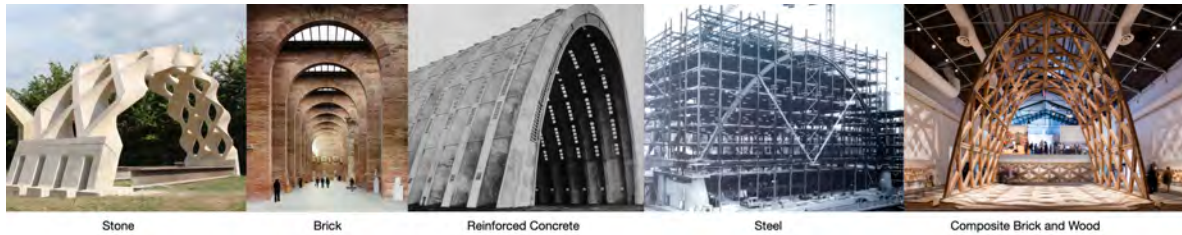


Figure E2.2: Examples of structural arches of various materials

## 2.2 Structural Behavior

In the mid-18th century, an Italian, Giovanni Poleni, determined the mechanism by which an arch carries load. His description of an arch formed by identical spheres, each one touching in a string forming an arch, transferring both the self weight of each sphere (a downward vector) and the force of the sphere above it (a vector also that is the resultant of the forces above), to the sphere below defined a line of vectors that came to be known as the *thrust line*. For the exact profile or shape of the thrust line, Poleni took reference from Robert Hooke's analogy of the hanging chain, that the shape of this line, representing only the dead weight of the spheres (or voussoirs) was exactly the inverse of a hanging string or chain, which is known as a *funicular shape*. Mathematically this shape is a catenary curve.

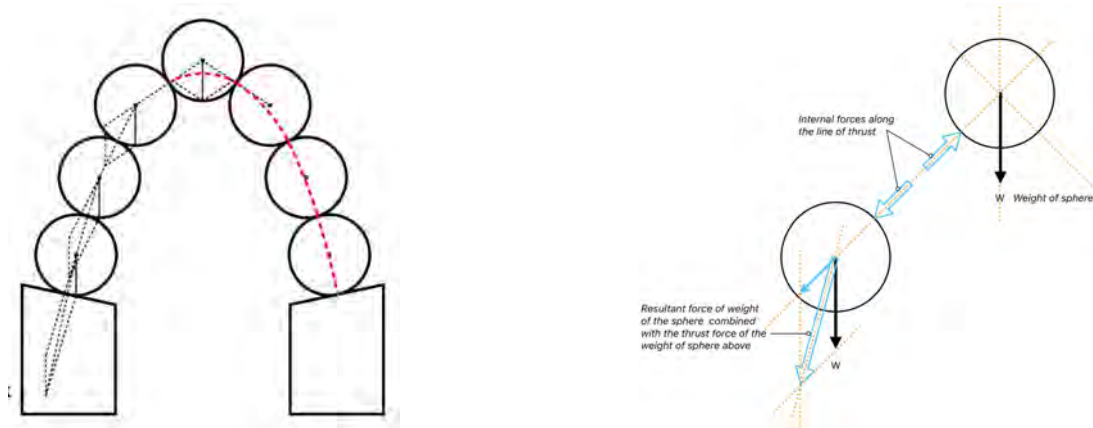


Figure E2.3: Poleni's spherical model describing the thrust line of an arch

The thrust line of an arch is determined wholly by the position and magnitude of the loads. It describes the *funicular* shape. If the centerline of the arch follows the thrust line determined by the loads, the arch will be purely compressive, that is, no bending moments will develop in the arch. This is the ideal condition and can only exist with the exact loads that define the thrust. Since the live loads in a building will vary over time, it is common practice to shape the profile of the arch to match the thrust line generated by the dead load of the arch and what it supports. The dead load is permanent and unchanging and is generally a large portion (sometimes as much as half) of the total load.

If an arch is not shaped to follow the thrust line of the loads it carries, the amount of bending moment developed in different sections of the arch, can be inferred from the amount of divergence between the arch profile and the thrust line. In masonry arches the presence of bending moments causes voussoirs in the arch to rotate and create gaps either on the *intrados* or the *extrados*, the inside or outside surfaces of the arch. These gaps were worrisome to the early builders of such arches and the emergence of four such rotations and gaps would lead to the collapse of the structure.

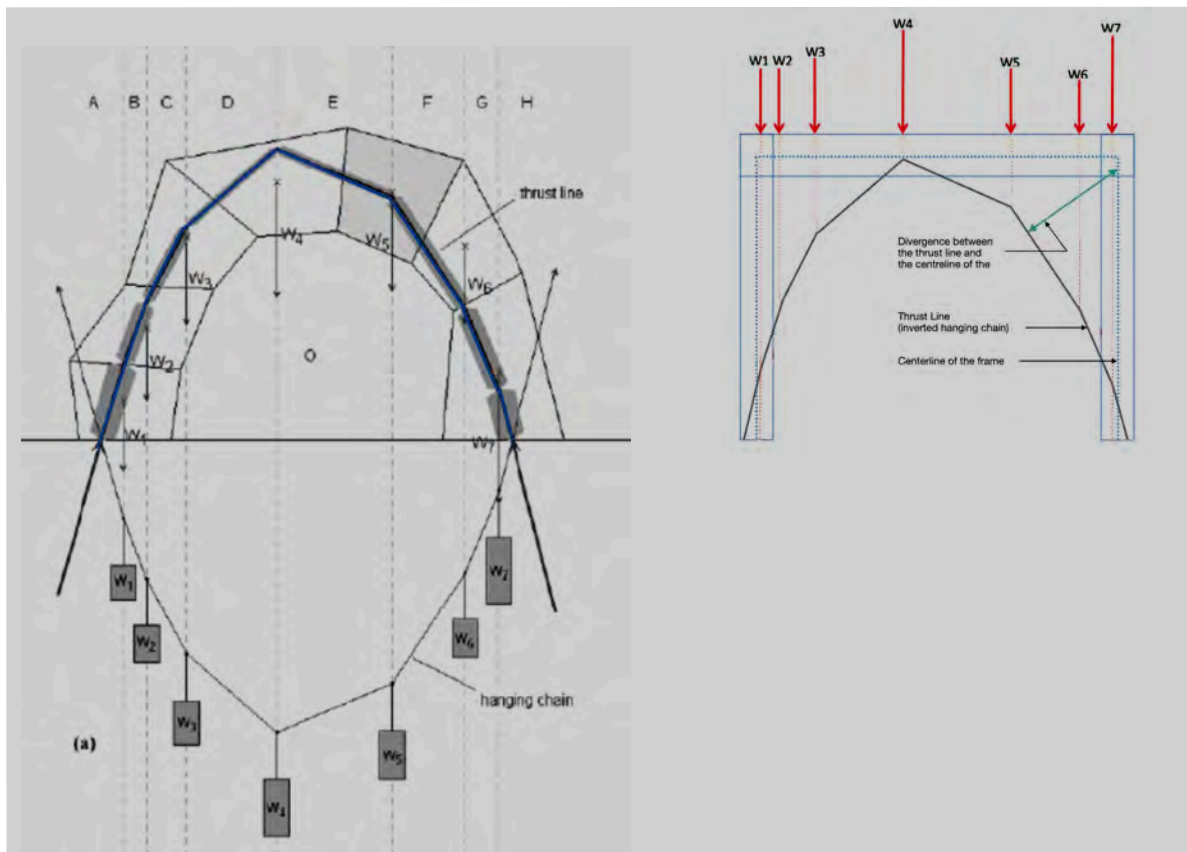


Figure E2.4: Thrust line superimposed on a rigid frame

Left: Irregular stone arch with trajectory of thrust line superimposed ("As Hangs the Flexible Line: Equilibrium of Masonry Arches", Block/DeJong/Ochsendorf. Nexus Network Journal, Vol.8, No.2, 2006.)

Right: The lines of action of the weights are identical to the lines of action in the stone arch, therefore the natural thrust line or *stress trajectory* will be the same. In this case the stress trajectory is mostly outside the frame structure. The amount of divergence from the centerline of the frame structure is an indication of the amount of *bending force* present in the frame.

In an arch with pin connection supports, the thrust line passes through the hinges and determines the reaction forces at the supports. The thrust of the arch at the pin has both a vertical and horizontal component (i.e., the thrust is never purely vertical or horizontal). In any symmetrical arch the vertical component is equal to one half the vertical loading on the arch. It is unaffected by the height or *rise* of the arch. The horizontal component depends on the angle of the thrust and hence the rise. The higher the rise, the lower will be the horizontal component of the thrust. Flatter arches with a lower rise will have larger outward thrust forces.

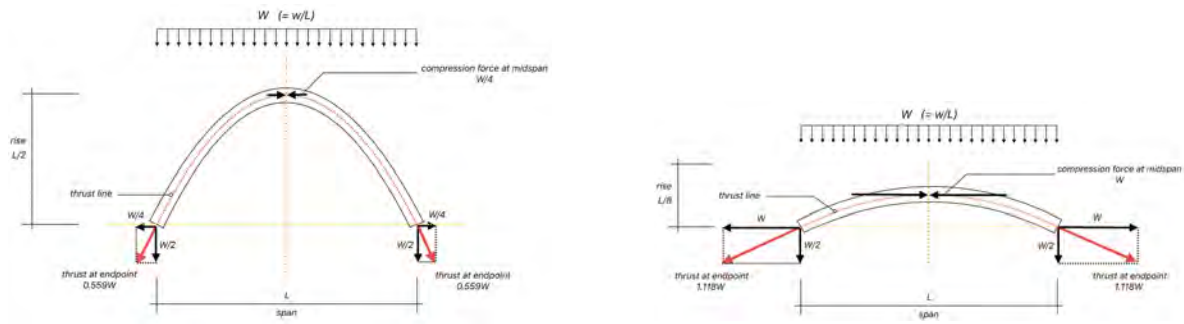


Figure E2.5: Arch thrust versus rise

### 2.3 Types of Arches: Fixed and Hinged Connections

Arches generally incorporate hinge connections in their design, partly because of construction advantages but also because of their impact on structural behavior. Theoretically, every joint between each voussoir in a stone arch is a simple hinge. Contemporary arches, however tend to incorporate a hinge at the midpoint of the span as well as at the supports. Hinge connections allow for some rotational movement which can accommodate thermal expansion, uneven settlement and load imbalances without developing additional stress generated by bending moments. On the other hand, fixed arches (fixed supports and no hinge joints in the span) are much stiffer and have less deflection under load. (See *Structures*, Schodek/Bechthold. Figure 5.34 p207)

### 2.4 Stability of Arches

Most arches are tall, relatively thin planar structures and are susceptible to both *lateral instability* (overturning) and *lateral buckling* (compression induced member buckling). In most buildings, arches that are used for structural support generally receive adequate bracing from attached building elements to prevent lateral buckling. To prevent overturning, arches must be braced against another arch or a structure that has lateral resistance. Otherwise the arch will require fixed support conditions *and* the cross section will become larger.

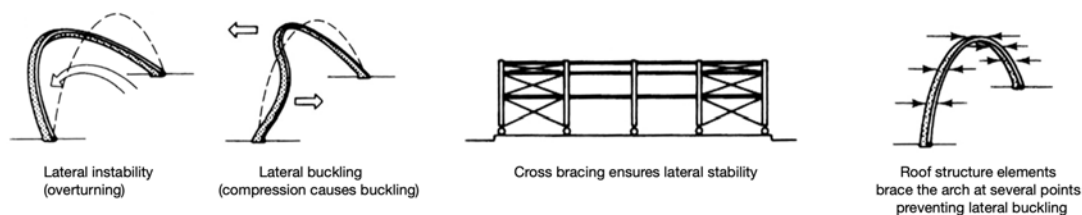


Figure E2.6: Stability of arches. *Structures*, Schodek/Bechthold. Figure 5.25 p197

### REVIEW QUESTIONS

- Is a “corbeled arch” a true arch? Describe the structural behavior of a corbeled arch.
- Sketch the funicular shape of an arch with various combinations of concentrated and distributed loads.
- What is the most efficient profile for an arch carrying a distributed load? What is the most economical rise to span ratio? 1:4, 1:6, 1:8 or 1:10?
- Explain the difference between “lateral buckling” and “lateral stability” in an arch.
- Where are the largest forces in an arch located?

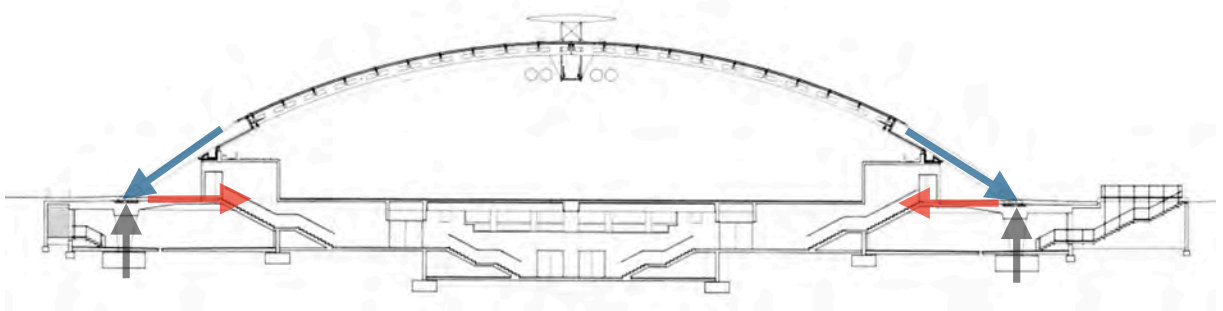
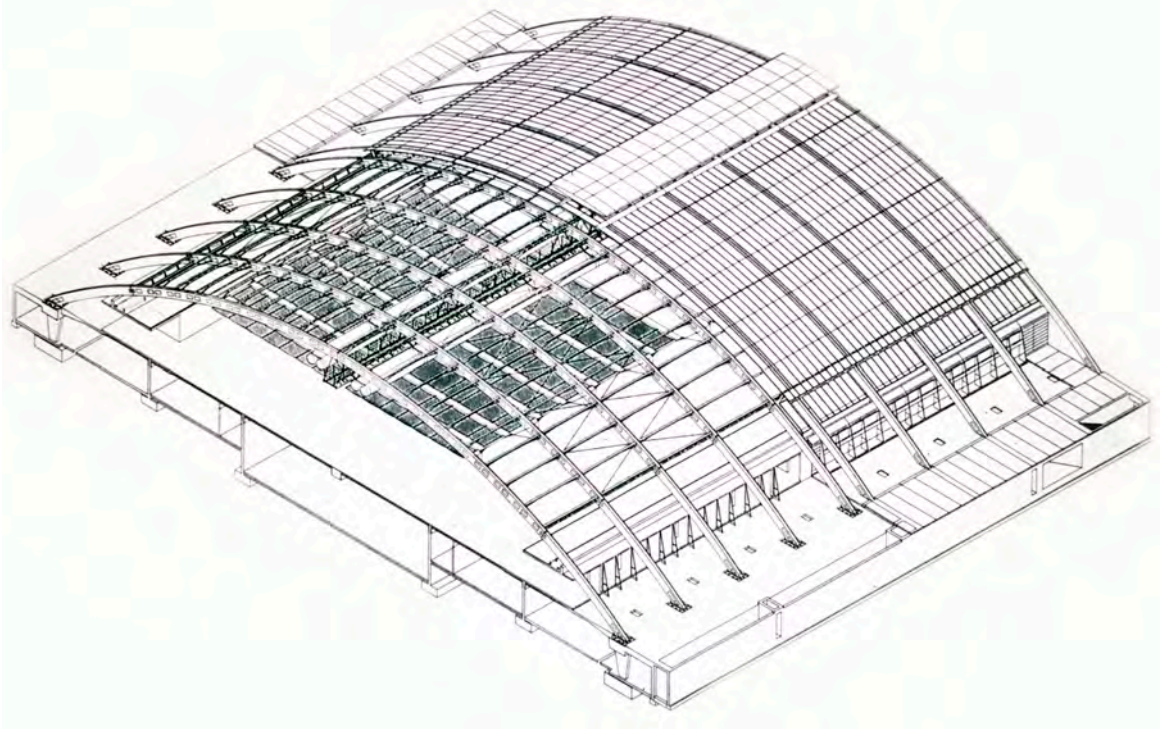


### Case Study: Design Center. Linz, Austria.

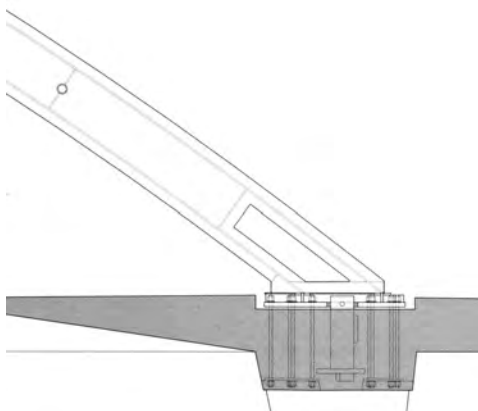
The Design Center in Linz is a large convention/exhibition hall designed by Thomas Herzog and Jorg Schrade (Herzog + Partner) and completed in 1994. The building is recognized for its technically advanced glazed roof system that incorporates a light reflecting grid within the double glazed roof panels. The transparency achieved by the glazing is achieved with an ultra-thin secondary framing system of propped steel T-beams spanning between the arches and supporting the glass frames of the roof. The main structure of the hall is a series of 34 parallel steel flat-arch box girders that span 73m.

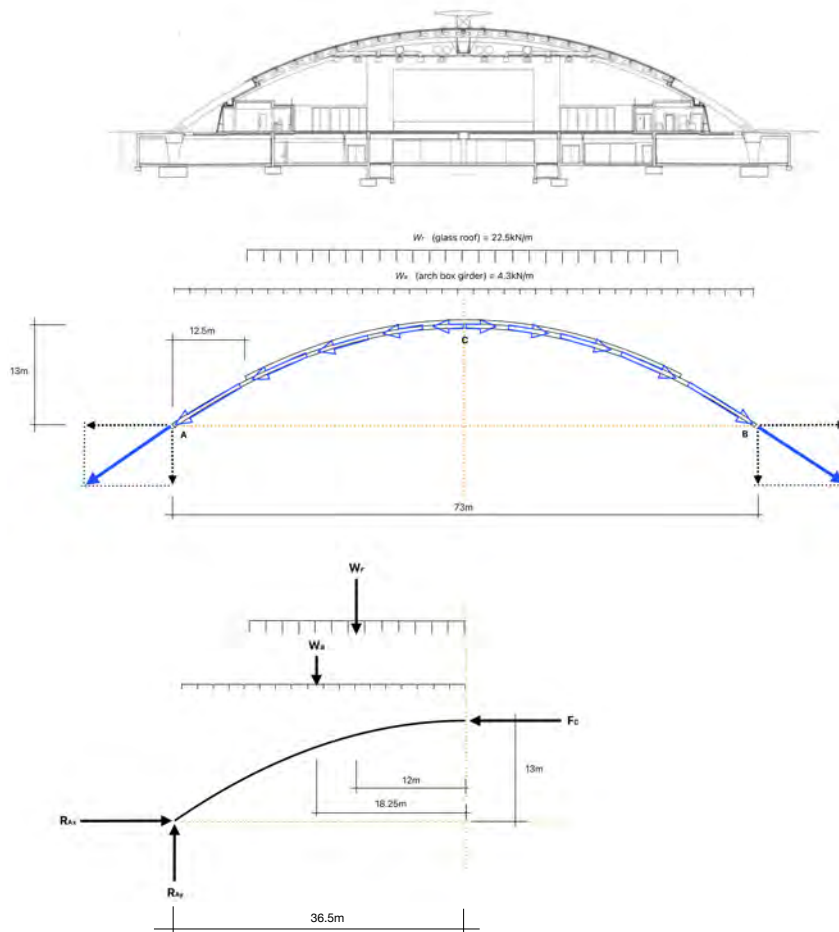






Outward thrust at the base of the arch is resisted by a horizontal steel tensile cable that connects the plate-ends of the arch across the entire width of the hall. These tension cables are placed in sleeves in the concrete of the main floor plate. The piers underneath the ends of the arch support only the vertical component of the thrust (the weight of the structure) and transmit this force to the footings beneath the lower level.





Calculate the thrust of the arch at its end support.

Area Loads:	Live load on roof (snow_Linz, Austria: ÖNORM B 1991-1-3)	1.5 kN/m <sup>2</sup> (approx 30psf)
	Dead Load: roof and secondary framing	1.0 kN/m <sup>2</sup> (approx 20psf)
	Dead Load of the arch girder (Total weight 36to)*	36,000kg x 9.807 = 353 kN

Line Loads	$w_r = (1.5 \text{ kN/m}^2 + 1.0 \text{ kN/m}^2) \times 7.2\text{m} =$	18.0 kN/m
	$w_a = (353 \text{ kN} / 73\text{m}) =$	4.84 kN/m

Resultant Loads	$W_r = 18.0 \text{ kN/m} \times (36.5\text{m} - 12.5\text{m}) =$	432 kN
	$W_a = 4.84 \text{ kN/m} \times 73\text{m} =$	353 kN

In the diagram above, to determine the thrust at support A, first determine the horizontal component of the thrust,  $R_{Ax}$ .

Note: For equilibrium of forces in the x direction,  $R_{Ax} - F_C = 0$  therefore,  $R_{Ax} = F_C$

$$\begin{aligned} \sum M_A = 0 \quad & (W_r \times 24.5\text{m}) + (W_a \times 18.25\text{m}) - (F_C \times 13\text{m}) = 0 \\ & (432 \text{ kN} \times 24.5\text{m}) + (353 \text{ kN} \times 18.25\text{m}) = (F_C \times 13\text{m}) \\ & \frac{(10,584 \text{ kN-m}) + (6,442 \text{ kN-m})}{13\text{m}} = F_C \end{aligned} \quad \quad \quad \mathbf{F_C = 1,310 \text{ kN} = R_{Ax}}$$

$$\begin{aligned} \sum F_y = 0 \quad & R_{Ay} - W_r - W_a = 0 \\ & R_{Ay} = 432 \text{ kN} + 353 \text{ kN} \end{aligned} \quad \quad \quad \mathbf{R_{Ay} = 785 \text{ kN}}$$

$$\text{Thrust at A} \quad R_{Ax}^2 + R_{Ay}^2 = R_A^2 \quad \quad \quad \mathbf{R_A = 1,528 \text{ kN}}$$

\* *Design Center Linz*. Thomas Herzog. Gerd Hatje, Stuttgart, 1994.

### 3.0 Tensile Structures: Suspension and Cable Stayed

A tensile structure is a span structure carrying load in tension over its entire length. Suspension and cable stayed are two types of tensile structures typically formed with flexible steel wire cables. A structure that is flexible under load will deform to a shape in which only tensile stress is present (no bending stress). These structures are called *funicular* and belong to a class of structures referred to as *form-active*.

#### 3.1 Formal Characteristics

The earliest form of suspension structures were footbridges made with hemp, a natural material used for making rope. Since the primary supporting element, the cable or rope, is flexible, the shape of the structure changes as moving loads (people) are supported along the span at different locations. The form or shape that a suspended tensile structure assumes depends on both the magnitude and the placement of the loads that it supports. Nonetheless, a family of funicular shapes for the same loading condition exists. Each is unique having a different *sag* (or *rise*, in the case the inversion, an arch). The difference is the length of the suspension cable.

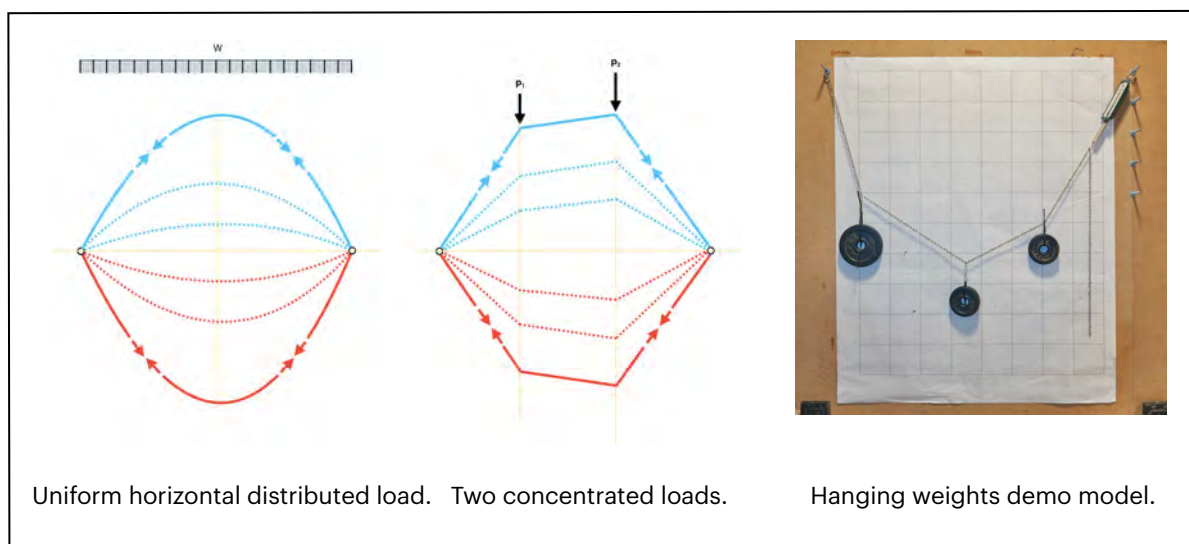


Figure E3.1: Family of funicular forms with distributed and concentrated loads.

Within the category of suspension structures, there are several different types.

- Parallel span systems. Most suspension bridges are of this type employing two main cables. Attached to the main suspension cables are *hanger* cables that carry the weight of the bridge deck and loads to the suspension cable. These are spaced equally and therefore the loading on the main cable is a horizontally distributed loading that results in the suspended cable assuming the shape of a parabola. A series of single curvature spans side by side can be used to form large roof areas. A second cable opposite in curvature to the first may be added for the purpose of stiffening against wind loads.
- Radial span systems. Analogous to a bicycle wheel, this form consists of a central tension ring, an outer compression ring, and cables in a radial configuration stretching between the two.
- Biaxial cable systems. These are cable grid nets with two opposing curvatures (*anticlastic*) which causes them to be naturally stiff. Geometrically they are hyperbolic paraboloid shapes.

- Cable trusses. A truss in which all the tension force members are flexible cables is referred to as a cable truss. The *tensegrity* cable 3D truss is a special case. In a tensegrity structure the compression elements, while not touching each other, are compressed by the tension cables. The system has been successfully used in large arena domes.

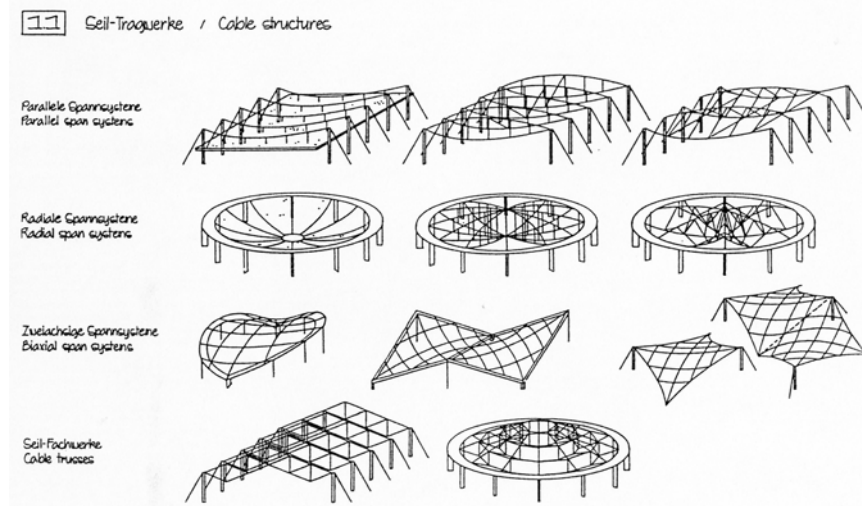


Figure E3.2: Suspension Structures. Various Forms. *Structure Systems*, Heino Engel. p60



Figure E3.3: Suspension Structures. Examples.

A *cable stayed* structure uses only straight cables to carry loads in tension directly from a point of support to a vertical structure such as a mast. The cable stay system shares the span load support with a rigid horizontal span member such as a beam or truss. In a bridge the cables extend from a mast or tower to support a bridge deck at regular distances determined by the span capability (flexural strength) of the deck structure. Each cable supports a contributory portion of the deck load despite the angle of the cable. The tension force in the cable however, will be greater in cables with a steeper inclination.

The cable stayed system is economical in roof design where a large clear span is desired. The cable supports including the mast and tieback cables are normally on the exterior of the building. This is an advantage for interior space planning but requires a larger footprint on the site for positioning the mast and tieback cables.



Figure E3.4: Cable Stayed Structures. Examples]



### 3.2 Structural Behavior

Flexible does not imply *stretchable*. A cable can bend easily but changing its length requires great force. Elongation of a cable is dependent on the material stiffness of the member. A material such as steel is extremely stiff ( $E$ , the elastic modulus is 210GPa. This is 7 times stiffer than hemp rope at about 30GPa). Tension structures are very efficient in carrying load as the full cross sectional area of the tension member, whether it be a cable or a non-flexible tension rod, is stressed uniformly. Furthermore, member stability issues such as buckling are not present.

Suspension structures that carry a uniform loading will be curved while those that carry only concentrated loads will have straight segments between the loads (neglecting the self-weight of the cable which is usually small relative to the loads being supported). The vertical distance from a line connecting the two ends of the cable to the lowest point of the hanging structure is called the maximum sag. The greater the sag, the smaller will be the tensile forces in the cable. As the sag of a suspended structure becomes less and less it approaches the horizontal but will never be able to be perfectly flat. This is the familiar “clothesline enigma”; it is impossible to apply enough tension to the ends of the clothesline to make it straight as even the weight of a single item of clothing forces some deflection of the clothesline and a high level of tension force in the clothesline itself.

The optimal sag to span ratio is 1:3 for uniform distributed loading and 1:2 for a single concentrated load. This results in lower tension force in the cable and not undue height in the mast support.

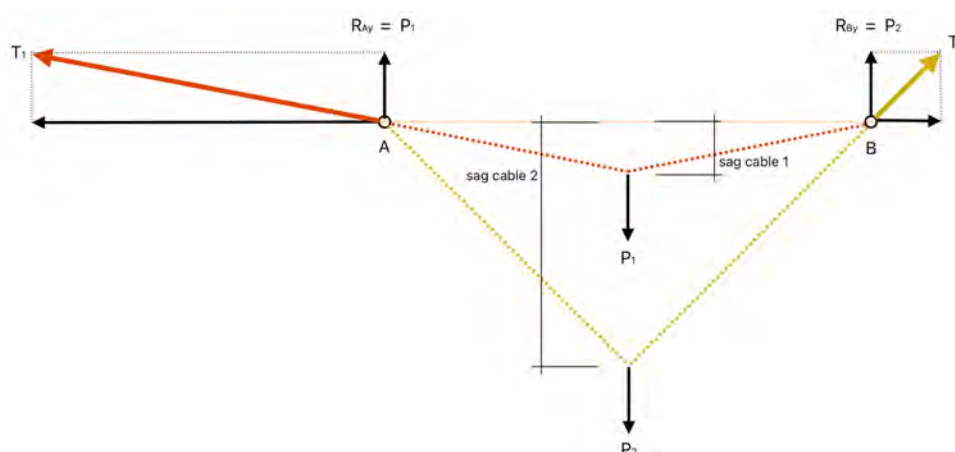


Figure E3.5: Thrust as a function of cable sag

A flexible suspended rope bridge may be an acceptable structure for pedestrians crossing a river gorge but for bridges that need to accommodate vehicles and buildings that support floors, a third element is required: a rigid deck or floor structure. The concept of the stiffened bridge deck resolved the issue of moving loads causing a distortion of the suspension cable. The rigidity of the stiffened deck distributes the localized load spreading it over a larger portion of the bridge and minimizing the deflection under the load itself as well as the corresponding distortion of the suspension cable.

Suspension structures will have large internal tension forces especially for longer spans with relatively shallow sags. Ratios of sag to span of 1/10 to 1/20 generate large tension forces that increase towards the ends. The largest tension force in a cable will be found in the portion of cable that has the steepest inclination from the horizontal. On account of this, the reaction forces at the ends of the suspension cable will be quite large and some

strategy to resist the inward horizontal thrust is required. There are several techniques, none of which have any effect on the forces in the main suspension cable.

- Tiebacks. One or more cables attached to the end of the suspension cable and anchored into the ground. The steeper the angle of this cable the greater will be the tension force in the tieback.
- Compression ring. The compression ring is very efficient as a means to neutralize the inward reaction forces of cables in a radial configuration. The tension forces, all pulling to the center, are resisted by the ring acting in compression like an arch. The length and radius of the ring cannot be easily deformed (shortened), especially if made of reinforced concrete or steel, thereby maintaining equilibrium with the inward forces.
- Tilted mast support. Leaning the mast support at an angle away from the span engages the mast in resisting a portion of the horizontal reaction force. In most cases the mast positioned in this way is usually provided with one or two tieback cables which also resist part of the horizontal reaction. In a few rare cases the mast support is treated as a tilted cantilever and must resist a large bending moment at its base that is generated by the horizontal component of the tension force at the top and the moment arm of the cantilever support.
- Buttress. A structure that acts as a buttress to counter the horizontal reaction force can take the form of a shear wall, a framed structure or a vertical truss. In each of these it is assumed that the base of the buttress is wide and together with the foundation in the ground creates a both a counter moment and a horizontal shear.
- Compression strut. A horizontal compression strut or plate element that connects the two ends of the suspension structure and opposes the tension thrust like a horizontal column. This element can be an integral part of the building such as a rigid diaphragm floor or roof element.

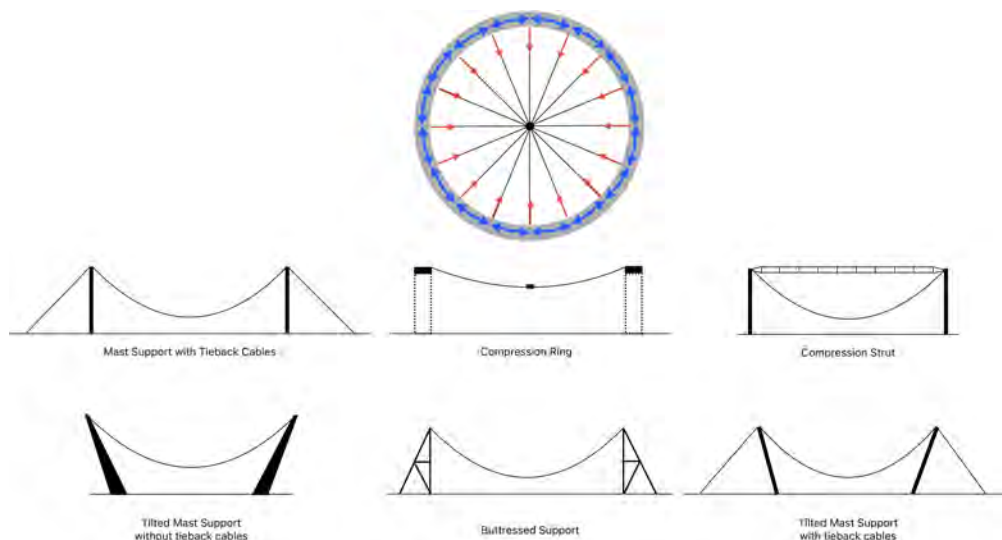


Figure E3.6: Resisting Horizontal Tension

### 3.3 Analysis of Suspension Structures

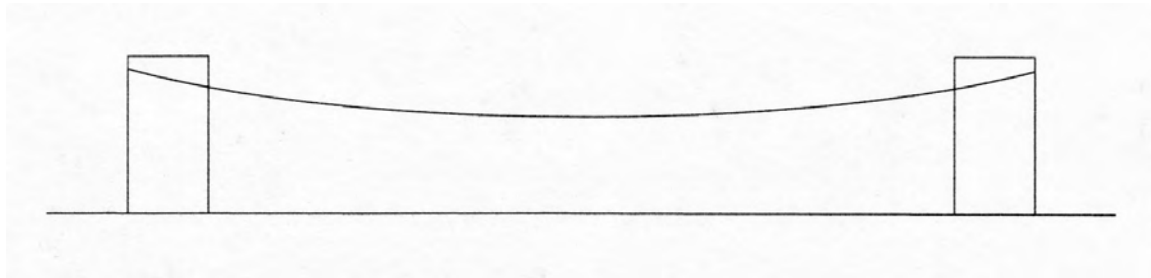
Finding the forces in a suspension structure of known configuration is a relatively straightforward process using the principle of equilibrium. As a funicular structure, there is no bending force present therefore every point of the structure may be considered as a hinge. By knowing the sag at one point of a suspension structure that supports concentrated loads, other sag points and the overall shape can be determined as the relative distortions are interrelated. For a suspension structure with horizontal uniform distributed loading the shape of the cable is a parabola. Knowing the maximum cable sag,  $h_{\max}$ , allows the internal forces in the cable to be determined.

### Case Study: Portuguese National Pavilion at Expo'98 in Lisbon, Portugal

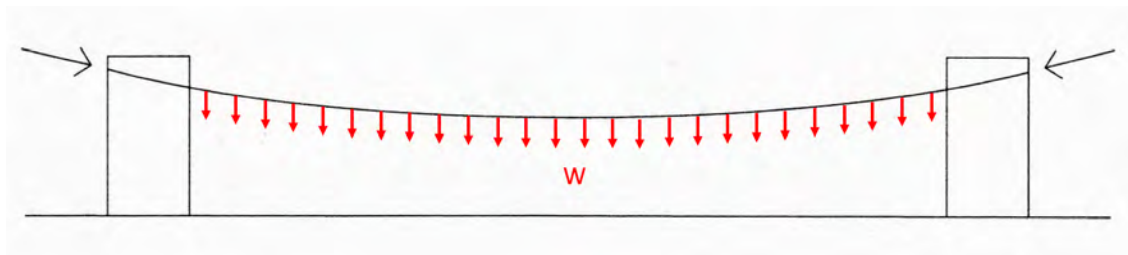
The Portuguese National Pavilion was created as a reception center for visiting heads of state attending the opening ceremonies of Expo '98. The design that architect Alvaro Siza conceived is a two part scheme: a modest two-story administrative and exhibition building attached to a grand covered entry court. The entry court has two monumental porticos on either end that support a 70m suspended concrete slab canopy. The curved concrete slab is threaded with postensioned steel cables that carry the weight of the canopy and attach to the reinforced concrete structure of the porticoes on either end. It is a funicular but rigid suspension structure.



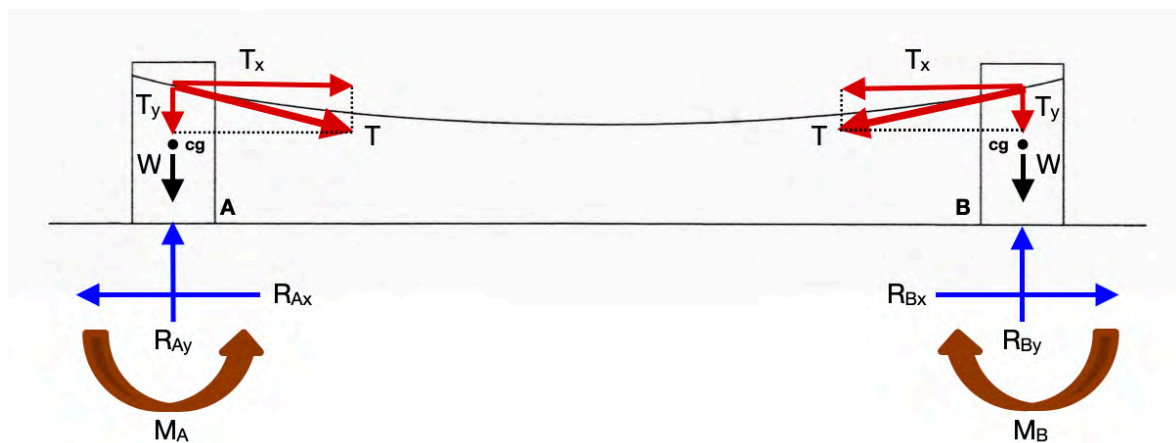
From top clockwise. Aerial view. The apparent visual thinness of the canopy contrasted with the monumental scale porticos that support it. View of portico with green tile surfaces. View of exposed steel cables at the edges of the slab.



Basic diagram of geometry of structure.



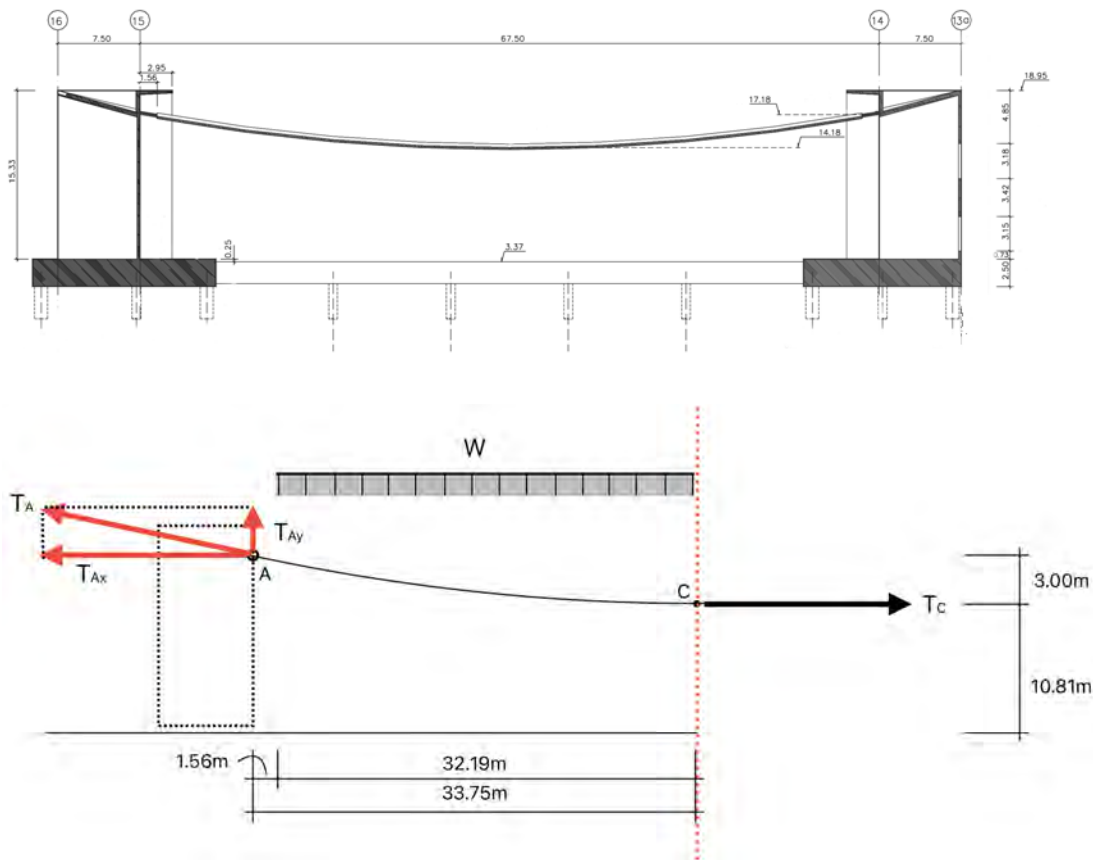
Loading diagram of roof structure. "W" (kN/m) represents the weight of a 1 m strip of the concrete and steel roof canopy distributed equally along curved roof surface between the supports.



Force diagram:  $T$  (red force vectors) represent roof cable tension force and x-y force components.  $W$  represents the weight of the pier wall support structure.  $R_{Ay}$  is the vertical reaction force at A, equal and opposite to  $W + T_y$  (the vertical downward force component of the cable tension force  $T$ ).  $R_{Ax}$  is the horizontal reaction force at A, equal and opposite to  $T_x$  (the horizontal component of the cable tension force).  $M_A$  is the reaction moment produced by the horizontal component of the cable tension force  $T$  times the moment arm (the perpendicular distance from the line of action of  $T_x$  to the base of the wall).



Calculate the forces  $T_A$  and  $T_C$  (at the end and center of span) of the suspension cables.



Loads	dead load of roof	(20cm lightweight concrete @1800kg/m <sup>3</sup> ) 20cm x 1800kg/m <sup>3</sup> x 9.807	= 3.5kN/m <sup>2</sup>
	reinforcing and post-tensioning steel cable*		= 1kN/m <sup>2</sup>
	live load on roof	(recommended* 0 - 1kN/m <sup>2</sup> )	Use 0.5kN/m <sup>2</sup>
	combined load		5kN/m <sup>2</sup>

\*Eurocode 1: Actions on Structures - Part 1-1: General actions - Densities, self-weight, imposed loads

Find tension force in cable

Cables are spaced approximately 1m on center. For a 1m wide strip of the roof,  $w = 5\text{ kN/m}$

$$\sum M \text{ about point A: } [(5\text{ kN/m} \times 32.19\text{ m}) \times (16.1\text{ m} + 1.56\text{ m})] - T_C \times 3.00\text{ m}$$

$$T_C = 160.95\text{ kN} \times 17.66\text{ m} / 3\text{ m} = \text{about } \mathbf{947\text{ kN}}$$

What is the tension force (reaction) at the endpoint A?

$$\sum F_x = 0 \text{ and } \sum F_y = 0: \quad T_{Ay} = W = (5\text{ kN/m} \times 32.19\text{ m}) = 160.95\text{ kN} \text{ and } T_{Ax} = -T_C = 947\text{ kN}$$

$$(T_A)^2 = (T_{Ax})^2 + (T_{Ay})^2 = (947\text{ kN})^2 + (160.95\text{ kN})^2$$

$$T_A = \mathbf{961\text{ kN}}$$

The values for  $T_A$  and  $T_C$  are large due to the sag to span ration:  $3 / 67.5$  or approximately  $1 / 22$ .

In a cable-stayed structure, the forces in the cable can usually be determined by the assessing the equilibrium at the endpoints of the cable. The angle of the tension force in the cable is known (axial force means that the line of action of the cable force is the angle of the cable itself) therefore only the vertical or horizontal component of the reaction force must be known. The magnitude of the vertical component of the cable tension force is equal to (but opposite in direction) the total vertical load acting at that point.

### 3.4 Stability of Tension Structures

Suspension structures, because of their lightness, are most susceptible to the force of wind. For this reason, their design must incorporate a strategy to resist movement and uplift caused by wind. Most suspension structures tend to be long-span roof designs. Wind passing over a building creates both positive and negative pressures on the roof. Movement caused by these wind pressures can be resisted in several ways.

- Weight. Although the cable supporting structure of a roof design is light, the cladding and secondary spanning elements can be intentionally made heavy so as to act as a kind of *ballast*, or weight that can resist uplift forces due to wind. This stabilizes the structure. In the past, poured concrete or precast concrete roof elements were used to achieve the necessary weight. Today there are other techniques such as hollow wood box sections filled with sand or gravel and used as secondary span structure.
- Double cable systems. The use of two cables, independent of each other, curving in opposite directions and separated by either compression struts or cable ties is common. If the wind is creating a downward force on the roof, the cable with the concave profile resists the force in tension. For uplift forces on the roof, the cable that is convex in profile resists the upward force. The double cable system has an inherent stiffness that also counteracts *flutter*. The cables are pre-tensioned to avoid the possibility of one of the cables becoming slack (without any tension force in it).
- Tieback cables. Tieback cables connect parts of the main suspension cable (that carries the roof loads) either directly to the ground or to a secondary suspension cable that is attached to the ground. The Brooklyn Bridge in NYC (1883) by John Roebling used a set of tieback cables to minimize vibration and flutter on the cable hangers (that support the bridge deck). The tieback cables connect to the top of the two bridge towers.

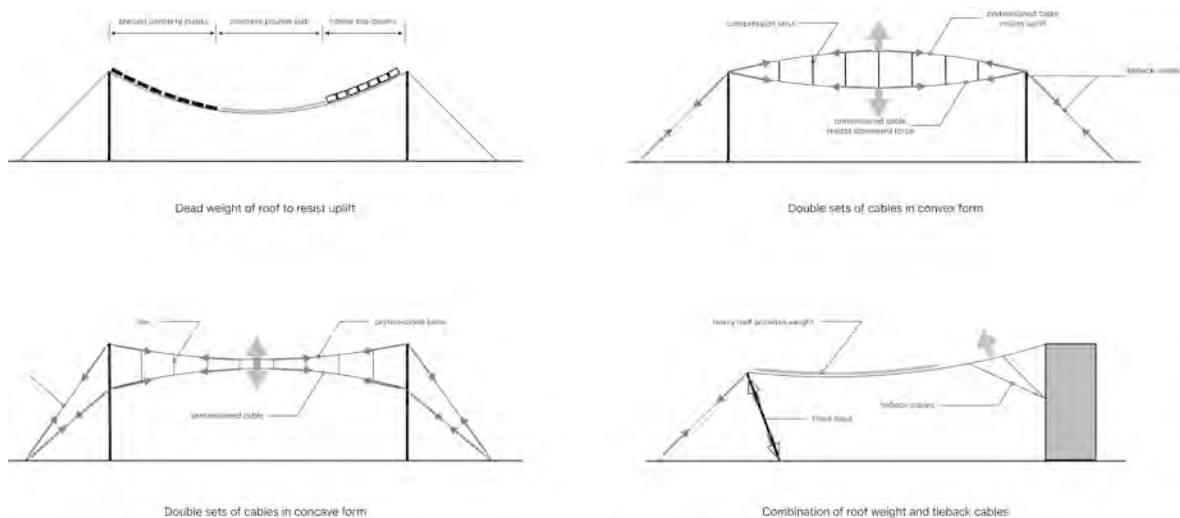


Figure E3.7: Counteracting Wind Force

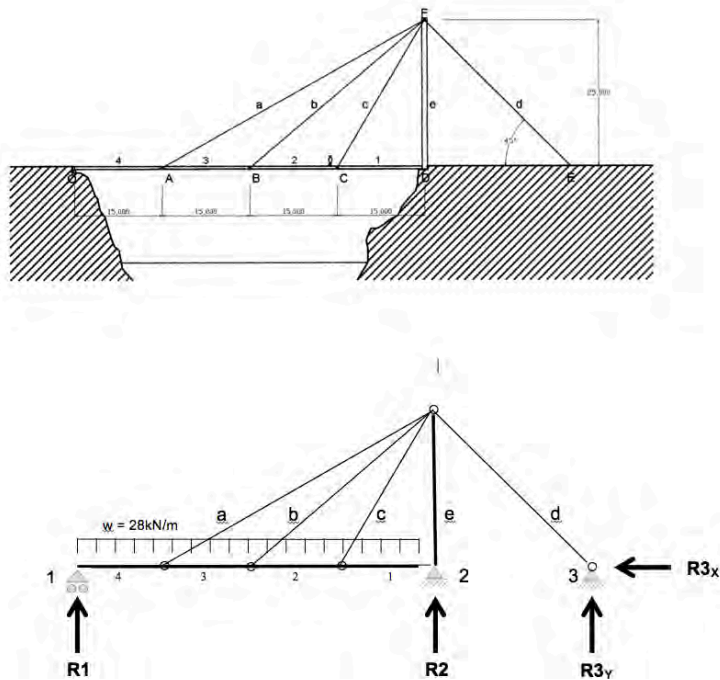
## REVIEW QUESTIONS

- Are all suspension structures funicular? Can a tension structure be funicular but not flexible? Explain.
- Why is a tension structure lightweight?
- External loads (gravity, wind, imposed loads) supported by any structure over a span creates an external moment on the structure. How does a cable suspension structure resist such a moment?
- A biaxial cable system obtains stiffness if it has an anti-clastic shape. True or false.
- Where is the maximum tension force in a suspension cable system located? At the point of maximum sag or at the end supports?
- What are some methods to resist the large tension forces at the ends of a suspension structure?
- Identify two methods of counteracting wind flutter in a suspension structure.

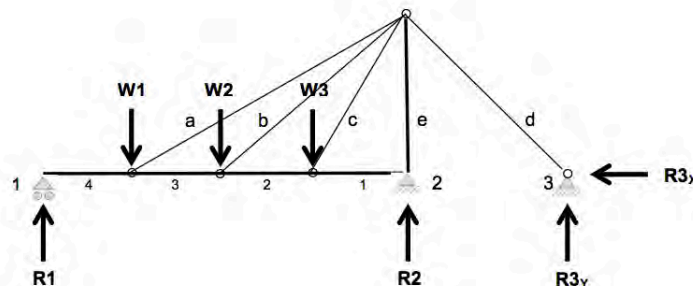
## Analysis of a cable stayed bridge

The small footbridge illustrated below is a cable-stayed structure. It has a rigid bridge deck that acts as a stiff beam spanning between points of support provided by the sloping cables, labeled a, b, & c. These cables transmit the weight of the bridge deck and its live loads to the top of the mast, from which the force is redirected into the ground by way of a single sloping guy cable, d. The mast, e, plays an important role in redirecting the forces as well as controlling their magnitude.

Draw a structural model diagram indicating all forces and reactions. Assume a total distributed load  $w$  (includes both live and dead gravitational loads) on the deck of  $28 \text{ kN/m}$  (this takes into account the width of the bridge deck). Also assume the mast base is fixed to the ground, the left hand bridge support is a simple bearing support, and the backstay cable (d) is anchored into the ground. Determine the tension forces in the stay cables and back-stay cable. Also find the compression force in the mast.

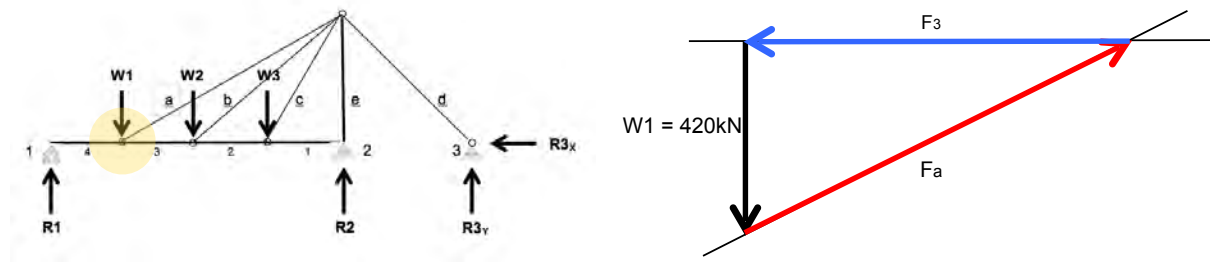


Step 1: Draw the bridge as a *structural model* diagram indicating support conditions, bridge deck loads and reactions. Label all structural elements.



Step 2: Determine the weight of the bridge deck supported by each of the sloping cables (a, b & c). The combined dead and live load of the bridge deck is  $28 \text{ kN/m}$  (given). Therefore each cable will support the *contributory* area of load of the deck, namely the length of deck on either side of the cable support to the midpoint of each deck segment. The weight may then be represented by a resultant concentrated load  $W$  ( $W_1 = W_2 = W_3$ ) =  $28 \text{ kN/m} \times (7.5 \text{ m} + 7.5 \text{ m}) = 420 \text{ kN}$





Step 3: Isolate each point of the bridge where the cable connects to the bridge deck as a *free body* diagram. Beginning at point A, the weight of the bridge deck supported by the cable a is represented by the resultant load vector  $W_1$  ( $= 420\text{kN}$ ).

Without knowing the angles of inclination of the cables, but knowing the length of the cables and the height of the mast, we can use the theorem of similar triangles to find the cable tension force  $F_a$ .

$$\text{Length of cable a} = L_a = \sqrt{(15\text{m} + 15\text{m} + 15\text{m})^2 + (25\text{m})^2} = 51.5\text{m}$$

The height of the mast (25m) corresponds to the vector  $W_1$  of the equilibrium triangle, Therefore, by similar triangles:

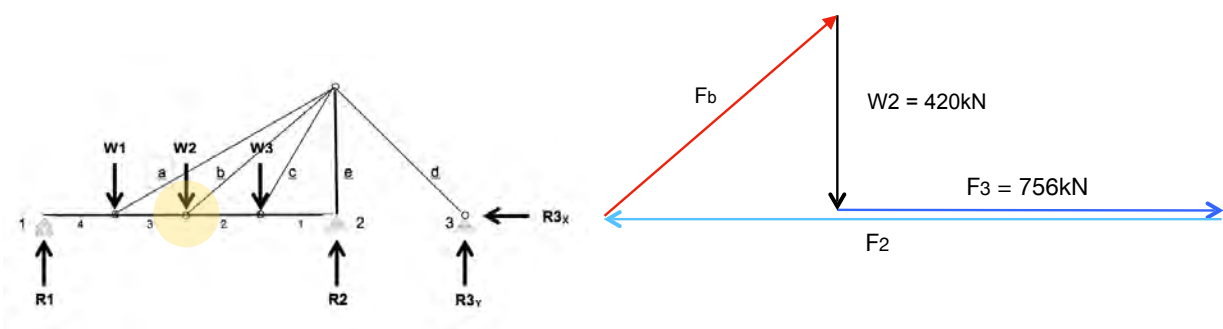
$$W_1 / 25\text{m} :: F_a / 51.5\text{m}$$

$$\text{and } F_a = 420\text{kN} \times 51.5\text{m} / 25\text{m} = 865\text{kN}$$

The cable tension imparts a compression force  $F_3$  into the bridge deck segment 3. This force is found also by similar triangles:

$$W_1 / 25\text{m} :: F_3 / 45\text{m}$$

$$\text{and } F_3 = 420\text{kN} \times 45\text{m} / 25\text{m} = 756\text{kN}$$



Step 4: For the middle cable b, the free body diagram of the point of connection has four vectors: the weight  $W_2$ , the compression force in segment 3, the compression force in segment 2 (note that the direction of a compression force is always into the joint. So the vector for  $F_3$  is now to the right while the vector for  $F_4$  is to the left.), and the tension cable force  $F_b$ . For equilibrium of the joint, these vectors must form a closed polygon as shown in free body figure above.

Now the triangle of forces is  $F_b$ ,  $W_2$ , and  $F_2 - F_3$ .

Again, referring to the similar triangle that represents the length of the cable  $L_b = \sqrt{(30\text{m})^2 + (25\text{m})^2} = 39\text{m}$

$$W_2 / 25\text{m} :: F_b / 39\text{m}$$

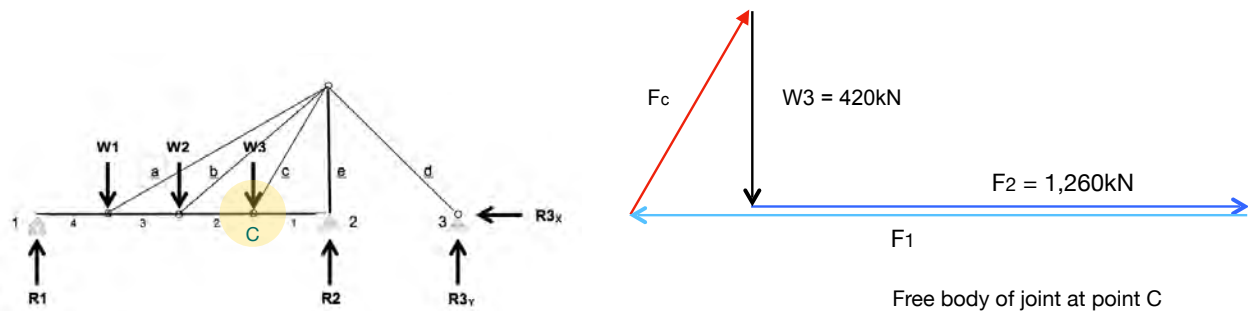
$$\text{and } F_b = 420\text{kN} \times 39\text{m} / 25\text{m} = 655\text{kN}$$

Again, the cable tension imparts a compression force  $F_2$  into the bridge deck segment 2. This force is found also by similar triangles:

$$W_2 / 25\text{m} :: F_2 - F_3 / 30\text{m}$$

$$\text{then } (F_2 - F_3) = 420\text{kN} \times 30\text{m} / 25\text{m} = 504\text{kN}$$

$$\text{and } F_2 = 504\text{kN} + F_3 = 504\text{kN} + 756\text{kN} = \mathbf{1,260\text{kN}}$$



Step 5: For the last cable c, the free body diagram of the point of connection (C) has four vectors: the weight  $W_3$ , the compression force in segment 2 ( $F_2$ ), the compression force in segment 1 ( $F_1$ , note that the direction of compression force vector for  $F_2$  is now to the right while the compression force vector for  $F_1$  is to the left.), and the tension cable force  $F_c$ . For equilibrium of the joint, these vectors must form a closed polygon as shown in free body figure above.

Now the triangle of forces is  $F_c$ ,  $W_3$ , and  $F_1 - F_2$ .

$$\text{Again, referring to Pythagoras to find the length: } \text{cable } c = \sqrt{(15\text{m})^2 + (25\text{m})^2} = 29.2\text{m}$$

$$\text{And similar triangles again to find the force: } W_3 / 25\text{m} :: F_c / 29.2\text{m}$$

$$\text{then: } F_c = 420\text{kN} \times 29.2\text{m} / 25\text{m} = \mathbf{491\text{kN}}$$

Again, the cable tension imparts a compression force  $F_1$  into the bridge deck segment 1. This force is found also by similar triangles:

$$W_3 / 25\text{m} :: F_1 - F_2 / 15\text{m}$$

$$\text{then } (F_1 - F_2) = 420\text{kN} \times 15\text{m} / 25\text{m} = 252\text{kN}$$

$$\text{and } F_1 = 252\text{kN} + F_2 = 252\text{kN} + 1,260\text{kN} = \mathbf{1,512\text{kN}}$$

Step 6: To find the force in the backstay cable, d, we can use the equation  $\sum F_x = 0$  at the point E (top of mast)

The cable forces each have an x-component acting to the left. The x-component of the  $F_d$ , the backstay will be equal to the sum and opposite in direction. This x-component of the  $F_d$  will also be equal to the reaction force,  $R_{3x}$ . To find the force in the cable, use the Pythagorean equation. (Ans. 2,138kN)

Likewise the compression force in the mast can be found by the equation  $\sum F_y = 0$  at point E. (Ans. 2,772kN)

Note: The compression force in the deck segment 1 will be resisted by a horizontal reaction at the support point 2 (not shown). Also note that the direction of the reaction forces at point 3, resisting the cable tension force in the backstay, will be opposite in direction to what is indicated.

## 4.0 Membrane Structures

Membranes are form-active surface structures that carry loads in tension. They are flexible structures and like cable suspension structures, their form is conditioned by the loads they carry. They comprise both tent and *pneumatic* membranes. They have been successful as temporary structures, however, with prestressing, it is possible to create more permanent enclosures.

### 4.1 Formal Characteristics

Membrane structures are best visualized with the use of soap bubbles. The thin film of a soap bubble represents in nature, the distribution of stress and the correct funicular form of highly complex shapes that are possible to achieve. Mathematically, the soap bubble forms the minimal surface required for enclosing a volume or space. Until the bubble is formed into a sphere, the soap film clings to it's generating edge, the hand-held forming "spoon" through which by blowing air, a soap bubble is released. A soap film can also be formed between the edges of any shape, polygonal or free form, either 2D or 3D.

Digital applications can now compute the minimum surface shapes for any configuration. Correct membrane forms are now easily achievable. This has led to recent experimentation not only in flexible membranes but also in free-form shell vaults that are funicular by analogy to the tension membrane.



Figure E4.1: Membrane Structures. Examples

Membranes gain rigidity from double curvature, that is, opposing curvatures in the perpendicular directions. Curved surfaces are classified in three categories referred to as *Gaussian* curvature. The first is the most basic: curvature in one direction such as a straight vault. The second has the same curvature in two perpendicular directions (*synclastic*). An example would be a dome. The third, referred to as negative gaussian curvature (*anticlastic*) has opposing curvature in perpendicular directions. An example is a hyperbolic paraboloid or saddle shape.

Of the three classes of curved surfaces, only the first one, singly curved surfaces is what we call *developable*; able to be formed from a flat, plane surface without stretching, shrinking or tearing. This has implications on construction and fabrication since building materials (like wood or steel) are generally flat. Concrete in the early 20c began to change this for shell structures and modern materials such as PVC coated polyester have done the same for membranes.

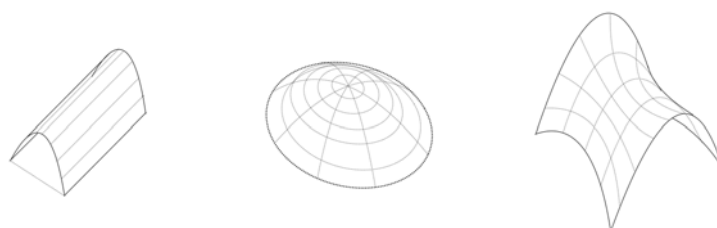


Figure E4.2: Types of curved surfaces: single curvature, double curvature and opposite curvature.

Pneumatic structures are membranes that sustain their shape from internal air pressure that must be strong enough to overcome both gravity and wind loads. Since they are pressurized with air, pneumatic structures form enclosed volumes. There are two basic types: air-inflated and air-supported

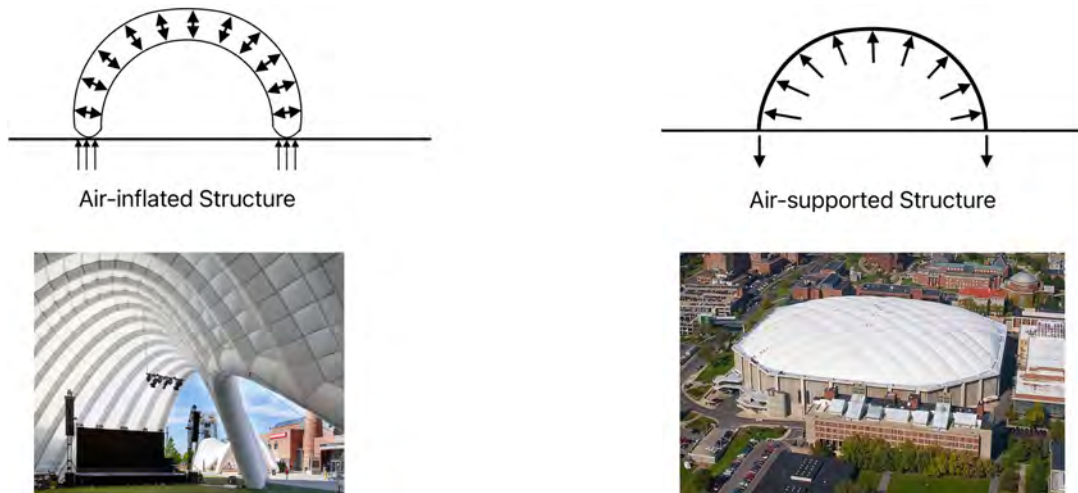


Figure E4.3: Pneumatic Structures

#### Air-inflated Structures

- No air seal required
- Openings can be placed more randomly
- Deployment easier
- Inflation rapid
- Higher pressure needed
- Greater acoustic and thermal control
- Extensions easy with segments

#### Air-supported Structures

- Air seal limits placement of openings
- Can span greater distances
- Easy to transport, quick erection
- Requires continuous air input
- Relatively low pressure required
- Low cost, short life span (7-10 yrs)
- Extensions are not convenient

Comparison of air-inflated and air-supported structures. Advantages and Disadvantages.

## 4.2 Structural Behavior

Membranes resist external loads through the development of in-plane tension forces in the surface of the membrane. These primary forces are similar to those found in a bi-axial cable net where the cables are stressed in perpendicular directions. They are two-way systems with large tension stresses accumulating along the edges of the membrane from where they are transmitted, usually with a cable, to corner points that redirect the forces to the ground with masts and tiebacks.

## 4.3 Stability of Membrane Structures

Because of their inherent lightness, primarily made from thin sheets of PVC-coated polyester with wire strands sometimes embedded, membranes are susceptible to deflection from wind, especially wind-induced *fluttering*. To counteract this action, membranes acquire stiffness from *anti-clastic* form; curvature in opposing directions. Hyperbolic paraboloid shapes, for example, are anti-clastic. Additionally, to prevent flutter, membranes and cable nets are prestressed in both directions (x and y) to prevent



compression forces from developing which would cause the flexible membrane to lose its tautness.

Pneumatic structures achieve stability as a result of *air pressure* that must be maintained within the enclosed double-skin volumes of the structure. This air pressure not only supports the structure over a span but also stiffens (pre-stresses) the membrane making it less susceptible to flutter caused by higher wind speeds.



Figure E4.4: Wind Effects on Pneumatic Structures

## REVIEW QUESTIONS

- Are membrane structures funicular?
- What are some differences between a cable net structure and a membrane structure?
- How do flexible membrane structures achieve stiffness? Can they carry loads?
- What is the span range of a pneumatic air-supported membrane structure?
- What does pre-stressing do for a membrane structure?
- Are membrane structures “permanent”? What are some reasons?
- What are the three categories of Gaussian curved surfaces?

## 5.0 Beams

*Beams* are horizontal linear span elements, deeper than they are wide. Loads placed perpendicular to the length of a beam, induce bending and curvature, which in turn create internal bending stress. These stresses are highly influenced by the size and configuration or shape of the section. Hence the beam is in the category of structures called *Section Active*.

### 5.1 Formal Characteristics

Beams are span structures that together with columns, create *frames*, the most common support system for buildings today. A beam can be made of any material that resists both tension and compression and has a required amount of stiffness, such that the beam element will not deform noticeably over the span. Common materials for beams are wood, metal (steel, aluminum), concrete, and fiber reinforced polymer (FRP). Glass has also been used in special circumstances. *Composite beams* are made with two or more materials. Reinforced concrete is an example. Another is the so-called *flitch beam* that combines a steel plate with one or more pieces of lumber.

There are different types of beams defined by their support condition. A *simple* beam is the most basic. It is a beam supported at either end, restrained against translational movement ( $x$  and  $y$ ) but free to rotate at the ends. If rotation is also constrained at the ends we refer to it as a *fixed* beam. If the support at one end is *fixed*, and the other end is unsupported, it is called a *cantilever*. Some beams are supported at several locations and are referred to as *continuous*. Various combinations of the above are possible such as a *propped cantilever* (one end fixed and a support located before the free end) and a *double cantilever* (a continuous beam with both supports pulled in from the ends of the beam).

Continuous beams may be extended over two, three or sometimes four bays. For continuous beams longer than two bays, a hinged connection is usually inserted because of length restrictions (beams can be made very long but they present problems in transport to the site). The location for the placement of hinges is often determined by the primary loading condition and the corresponding moment diagram. It is desirable to place the hinge at a point where the bending moment is small or zero. This allows the connection to be a simple hinge connection. A special case for the continuous beam is the concept called the *Gerber Beam*. It is a combination of two propped cantilever beams with a simple beam spanning between the ends of the cantilevers.

Most beams are linear extruded shapes, that is, they have a constant section along the longitudinal axis. A more efficient beam can be formed by altering the profile and/or section of the beam in response to the bending moment.

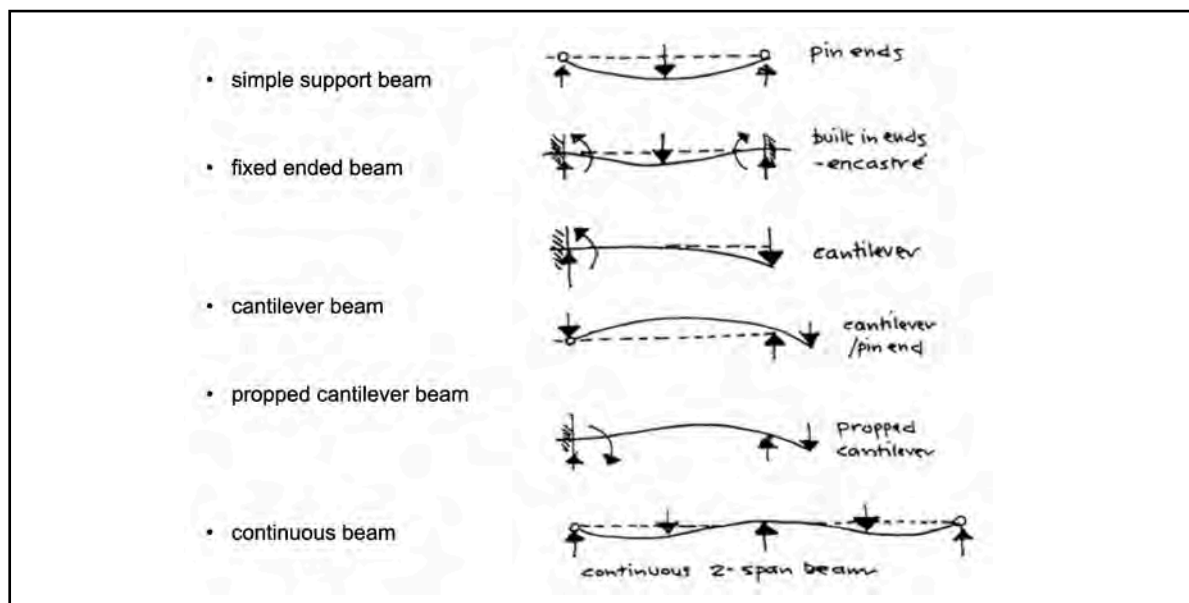


Figure E5.1: Types of Beams. *Tony Hunt's Structures Notebook*, Tony Hunt. P44

## 5.2 Structural Behavior

Beams are designed to carry loads that are applied perpendicular to the span. A beam-like element that resists only axial force loads (parallel to the length of the beam) is referred to as a strut or a tie member. If the loads are placed perpendicular and along the centerline of a beam, they induce bending and curvature in the beam. They also produce internal shear forces that act up and down on the section in an attempt to shear or break the beam. If placed off-center the loads will additionally induce a torsional moment causing the beam to twist or rotate about its longitudinal axis.

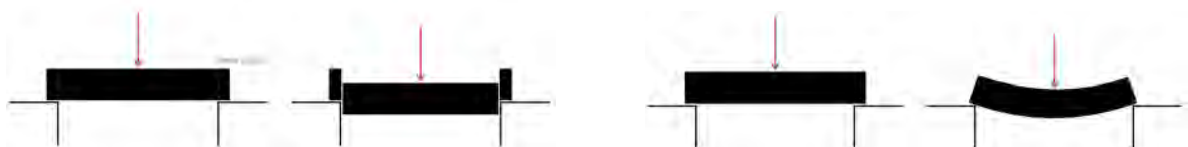


Figure E5.2: Beam Shear and Bending Moment

In most beams under gravity loads, the curvature will be concave upward, meaning that the beam will bend downwards. This bending of the beam (which is very slight and rarely visible) causes shortening of the length of the upper portion of the beam and an extension of the length of the bottom portion of the beam. The amount of bending stress that is generated is proportional to amount of shortening and lengthening of the material in the beam.

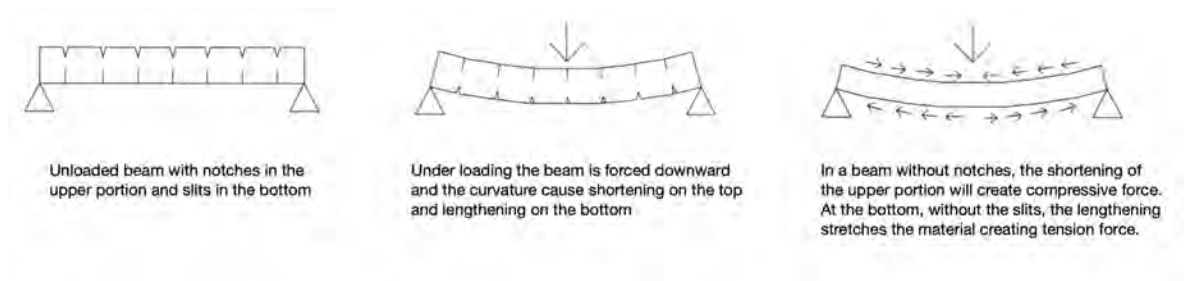


Figure E5.3: Beam curvature and bending stress

Shear and bending stress in a beam is more complex than axial stress because the intensity of the stress varies across the section. For vertical shear stress (acting in the plane of the section) the magnitude of the stress varies from  $v = 0$  at the top and bottom edge of the beam section to a maximum value at the center. Bending stress is the opposite with the maximum values of  $f_b$  at the top and bottom edge, and  $f_b = 0$  at the center or midpoint of the section. For sections that are not symmetrical, the value of  $f_b = 0$  will occur at the position of the neutral axis which corresponds to the centroid of the section.

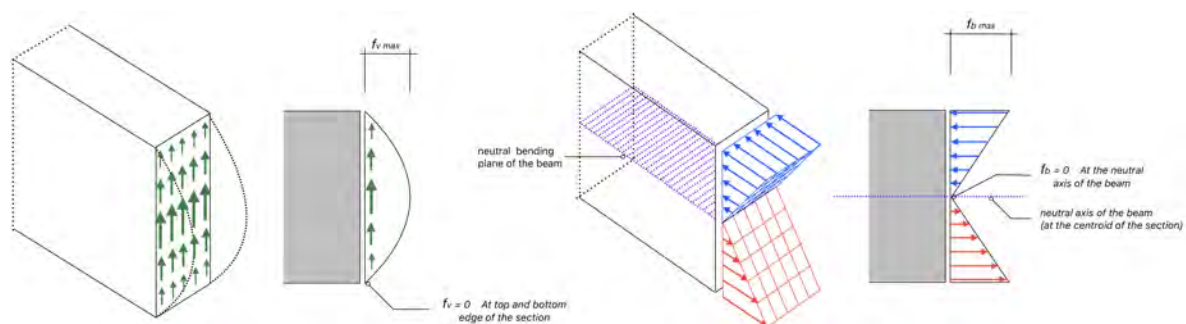


Figure E5.4: Beam shear and bending stress

Equations can be derived for the finding the maximum shear and bending stress in a beam. For maximum shear stress (at the center of the section) the equation is:

$$f_{v \text{ max}} = V / A$$

V is the shear at that place on the beam, A is the cross sectional area.

For bending stress, the equation is:

$$f_{b \text{ max}} = M c / I$$

M is the moment at that place on the beam, c is the distance from the centroid to outermost edge of the section, and I is the moment of inertia of the section.

### 5.3 Beam Analysis

Beam analysis means finding the internal forces acting on a beam and the location and magnitude of the maximum shear force and bending moment. Values can be determined at any point using the equations of equilibrium and these are plotted graphically along a line representing the length of the beam. These plots are referred to as *shear and bending moment diagrams*. (See Section 3.4 of Part B) One could argue that to an engineer, the moment and shear diagrams are as revealing as a musical score is to a composer.

### 5.4 Beam Design: Section and Profile

A beam is more efficient if it is deeper than it is wide. This is because bending is the primary stress in beams and a deeper beam has more bending resistance with the same amount of cross sectional area. For solid rectangular sections a proportion of 1:1.75 is economical. Too square (1:1) and material is not being used efficiently. Too deep (>1:4) and stability issues begin to compromise efficiency.

The process of beam design or selection begins with an analysis of the forces generated by the loads. This includes the *reaction forces* at the supports. Maximum shear and bending moments are found; their position indicated on the shear and moment diagrams. In a beam, the bending stresses are generally more critical than the shear stress, therefore a beam size and configuration will be determined primarily by the maximum bending moment. To determine the maximum bending stress,  $f_{b \max}$ , a trial section must be assumed. The values of  $c$  and  $I$  for the trial section will be used to calculate the stress. If that stress exceeds the *allowable stress* specified for that material (ordinary steel is  $F_A = 155 \text{ N/mm}^2$ ) then another section, with a larger  $I$  (moment of inertia) must be tried.

To simplify the process, one can use charts that list standard sections and provide the dimensions and properties for the various common shapes of major materials (steel, wood, aluminum, frp, and precast concrete). The equation for the maximum bending stress above can be revised to find the required moment of inertial (  $I$  ) for a given maximum stress:

$$I = M c / f_{b \max}$$

Since the distance  $c$ , although not yet determined, is  $d/2$  for a symmetrical section, a further simplification uses  $S$ , referred to as the Section Modulus (aka “Elastic Modulus”), and defined as  $I / c$  :

$$S_{\text{req'd}} = M / f_{b \max}$$

Once the required Section Modulus is determined, a beam section that has a Section Modulus equal to or greater may be selected.

A sample beam design problem together with an example of a universal steel beam chart is provided below.



## Beam Design

When we speak of *designing a beam* we are referring to the process of selecting the best beam that meets the criteria of safety and economy. Safety requires that the beam is designed to support all foreseeable load conditions and meet the three basic requirements of *strength, stability and stiffness*.

To meet the **strength** requirement, the beam must be designed not to fail due to stress. In a beam, the principle stresses that must be considered are bending and shear. The design also considers a *factor of safety* in determining the limits of stress under the given loads. Based on the span and the loads, the maximum bending moment and shear forces are found. To find the maximum stresses, the section of the beam and its important dimensional properties ( $A$  and  $I$ ) must be considered. Since these are not known until the beam is designed, a *trial and error* process is employed. A beam that has a section close in size to the desired beam is selected and then tested. Based on the outcome of the analysis, another beam section is chosen that more closely fits the sectional properties as determined in the analysis.

Beams tend to be unstable if they are deep and very thin. The design of a beam must check the proportions of the beam section and determine whether *bracing* may be required to insure **stability**. Guidelines are provided that take into account the end supports of the beam and its relative proportions of width and depth. In general, most beams in practice are braced by the floor or roof framing that they support.

The third design requirement is **stiffness**. Beam stiffness prevents excessive deflection. Even though a beam may be safely designed resist stress without failing, it may not be stiff enough. A relatively small amount of deflection can cause a beam to damage non-structural elements that are attached to it. Also, a beam that has excessive deflection may appear to be weak or unstable and thus un-reassuring to building inhabitants.

Each of these three conditions; insufficient strength, instability or excessive deflection must be evaluated in a beam design. In addition, beams must be designed to be as **economical** as possible. Usually this is accomplished by using the least amount of material required. Efficiency is achieved by adjusting the profile and the configuration of the section so that the beam uses its material in the most effective way possible. Choosing a section with the largest possible moment of inertia and shaping the profile of the beam to reflect the moment diagram are two standard techniques for increasing the performance and hence, the efficiency of a beam.

---

Exercise: Design a beam for a given span and loading condition.

The span is 8m and the beam will be simply supported. The beam will support a floor with a distributed load of 30kN/m based on live and dead load requirements and the contributory area.

- 1) Choose the type of beam.

Material choice and span (see Figure A1.7 p12 Section E: Approximate span ranges). For a span of 8m there are a few choices that are economical (in the middle of the range for the type). For example:

- Laminated wood beams
- Timber box beams
- R/C beams
- Pre-cast channels
- Universal steel beam (wide flange)
- Open-web steel joists

For this example we will choose a steel Universal Beam (wide flange).

- 2) First determine the maximum bending moment on the beam. This will determine the choice of section.

For a simply supported span, the  $M_{\max}$  will be  $wl^2 / 8$ . So  $M_{\max} = (30\text{kN/m}) (8\text{m}^2) / 8 = 240\text{kN-m}$

- 3) Determine the required Section Modulus.

$$S_{req'd} = M_{max} / f_b \text{ allowable} = 240 \text{ kN/m} / 155 \text{ N/mm}^2 = 1,548 \text{ cm}^3$$

\* (Use 155 N/mm<sup>2</sup> Allowable stress for Grade 43 Mild Steel)

- 4) Find a universal beam section with a section (elastic) modulus  $\geq 1,548 \text{ cm}^3$

In Table BS4 Part: 1993 (British Steel Standards) below we find a section **457x152x82** with an elastic modulus of **1571 cm<sup>3</sup>**. ( $> 1,548 \text{ cm}^3$ ). ✓

- 5) Shear will rarely be critical in a steel simply supported beam. No check required.  
6) We should check deflection.

In general, guidelines for required depth will usually indicate whether a problem exists.

A conservative span/depth ratio for a universal beam of 8m span is about 20.

For the beam 457x152x82 the depth is 465.8mm.

This satisfies the approximate ratio of  $8,000 \text{ mm} / 20 = 400 \text{ mm} \leq 465.8 \text{ mm}$ . ✓

However, if we wish to be more precise, we use the deflection formula:  $\Delta_{max} = 5wL^4 / 384 EI$

Substituting  $E = 205,000 \text{ N/mm}^2$  and  $I_x = 36,215 \text{ cm}^4$  we get a deflection of:

$$5 (30 \text{ kN/m})(8,000 \text{ mm})^4 / (384) (205,000 \text{ N/mm}^2)(36,589 \text{ cm}^4) \sim \mathbf{21.5 \text{ mm}}$$

Is this ok? Code usually requires a maximum deflection of  $L/240$  for combined load.

$$8,000 \text{ mm} / 240 = 33.3 \text{ mm} \quad 21.5 \text{ mm} \leq 33.3 \text{ mm} \quad \text{Therefore ok.} \quad \checkmark$$

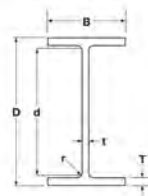
- 7) Stability. Since the beam will support flooring it will be braced by the purlins that support the floor structure.

Therefore no problems with stability should exist.

Structural Sections to BS4: Part 1: 1993 and BS EN10056: 1999

## Universal beams

Dimensions and properties to BS 4: Part 1: 1993



Designation	Mass per m	Depth of Section	Width of Section	Thickness of		Root Radius	Depth between Fillets	Area of Section	Second Moment Area		Radius of Gyration		Section (Elastic) Modulus		Plastic Modulus	
				Web	Flange				Axis x-x	Axis y-y	Axis x-x	Axis y-y	Axis x-x	Axis y-y	Axis x-x	Axis y-y
	kg/m	mm	mm	mm	mm	mm	mm	cm <sup>2</sup>	cm <sup>4</sup>	cm <sup>4</sup>	cm	cm	cm <sup>3</sup>	cm <sup>3</sup>	cm <sup>3</sup>	cm <sup>3</sup>
457x191x89	89.3	463.4	191.9	10.5	17.7	10.2	407.6	114	41015	2089	19	4.29	1770	218	2014	338
457x191x82	82	460	191.3	9.9	16	10.2	407.6	104	37051	1871	18.8	4.23	1611	196	1831	304
457x191x74	74.3	457	190.4	9	14.5	10.2	407.6	94.6	33319	1671	18.8	4.2	1458	176	1653	272
457x191x67	67.1	453.4	189.9	8.5	12.7	10.2	407.6	85.5	29380	1452	18.5	4.12	1296	153	1471	237
457x152x82	82.1	465.8	155.3	10.5	18.9	10.2	407.6	105	36588	1185	18.7	3.37	1571	153	1811	240
457x152x74	74.2	462	154.4	9.6	17	10.2	407.6	94.5	32674	1047	18.6	3.33	1414	136	1627	213
457x152x67	67.2	458	153.8	9	15	10.2	407.6	85.6	28927	913	18.4	3.27	1263	119	1453	187
457x152x60	59.8	454.6	152.9	8.1	13.3	10.2	407.6	76.2	25500	795	18.3	3.23	1122	104	1287	163
457x152x52	52.3	449.8	152.4	7.6	10.9	10.2	407.6	66.6	21369	645	17.9	3.11	950	84.6	1096	133
406x178x74	74.2	412.8	179.5	9.5	16	10.2	360.4	94.5	27310	1545	17	4.04	1323	172	1501	267
406x178x67	67.1	409.4	178.8	8.8	14.3	10.2	360.4	85.5	24331	1365	16.9	3.99	1189	153	1346	237
406x178x60	60.1	406.4	177.9	7.9	12.8	10.2	360.4	76.5	21596	1203	16.8	3.97	1063	135	1199	209
406x178x54	54.1	402.6	177.7	7.7	10.9	10.2	360.4	69	18722	1021	16.5	3.85	930	115	1055	178
406x140x46	46	403.2	142.2	6.8	11.2	10.2	360.4	58.6	15685	538	16.4	3.03	778	75.7	888	118

## 5.5 Beam Stability and Deflection

In addition to satisfying shear and bending stress requirements, a beam design must meet deflection criteria specified by the building code. For typical beams the maximum deflection under imposed loads is the span/200. For example, if the span is 8m, the specified maximum deflection of the beam should be  $\delta < 8000\text{mm} / 200 = 40\text{mm}$ . If the beam is finished with plaster or other brittle material, then the code max  $\delta$  is span/360. For a cantilever beam, the maximum  $\delta$  is limited to the length of the cantilever/180.

To see if the proposed beam selection meets the criteria, the predicted deflection must be calculated. Deflection formulas are provided in most appendices of structural design books. (See Figure 5.5: Typical V-M Diagrams of Beams in Part B). Note that the formulas have a pattern: Load ( $p$  or  $wL$ )  $\times L^3$  (span) /  $E I$ . Multiplied by a numerical constant (e.g.,  $5/384$ ).

The formula tells us that:

- As the load on the beam *increases* the deflection *increases linearly*.
- As the span *increases*, the deflection *increases exponentially* ( $2 \times L = 8 \times \delta$ ).
- As  $E$ , the elastic modulus *increases*, the  $\delta$  *decreases inversely* (linear).
- As  $I$ , the moment of inertia *increases*, the  $\delta$  *decreases inversely* (linear).

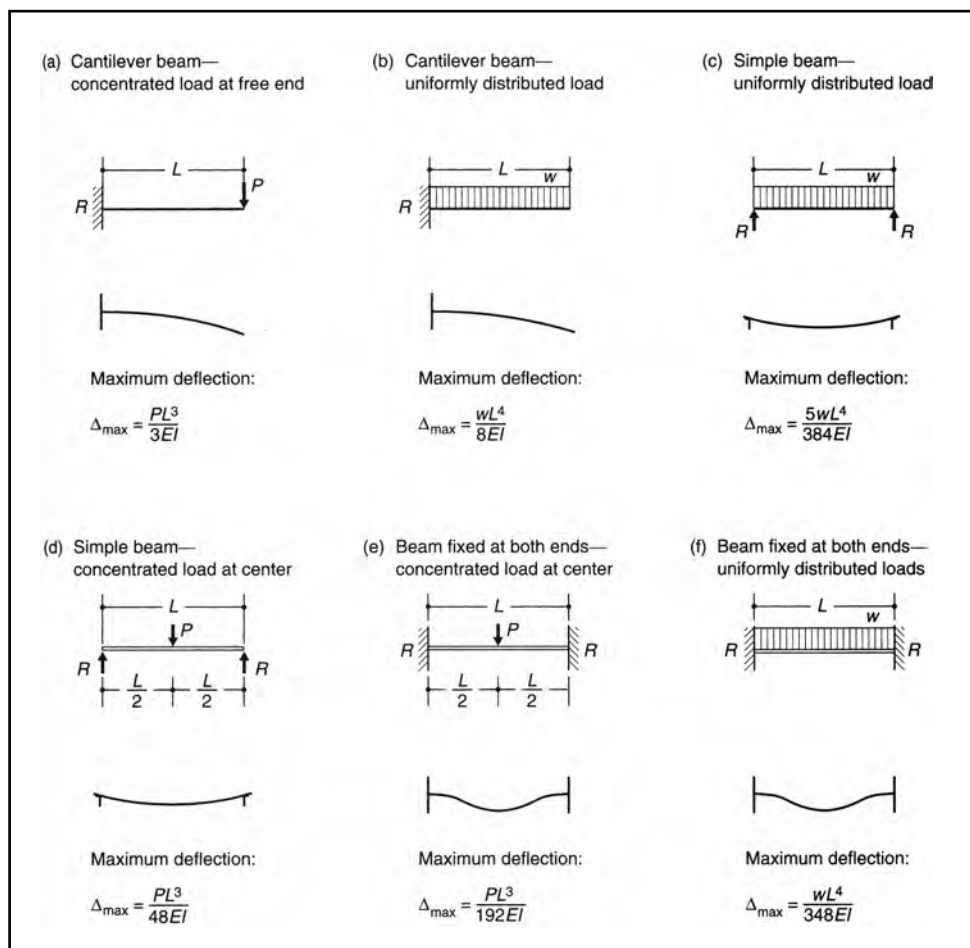


Figure E5.5: Deflections in beams. *Structures*, Schodek/Bechthold. Figure 6.24 p241

If the depth of a r/c concrete beam or slab has a span/effective depth ratio less than or equal to what is specified in the Code of Practice for the Structural Use of Concrete 2020

Ed., then there is no need to check deflection by calculation. Table 7.3 gives values. Similar depth ratios can be found in the Code of Practice for the Structural Use of Steel 2021 Ed. These values will tend to be conservative.

## 5.6 Trussed or Cable-supported Beams

Neither a beam nor a truss, the trussed beam is a beam stiffened with the use of a compression strut and tension members (rod, cable or steel section). A single strut trussed beam is basically an inverted King Post truss. The primary differences between a truss and a trussed beam are that the beam is usually stiffer than the corresponding upper chord member of a truss, it is continuous over the entire span, and it is capable of supporting heavy distributed vertical loads. The beam will develop compressive force in response to the tension forces of the cables and will also develop bending moments. The degree of the compressive force in the beam as well as the bending moments depend on the stiffness of the tension cables and the height of the compression strut. Stiffer cables produce higher compressive force and lower bending moments.

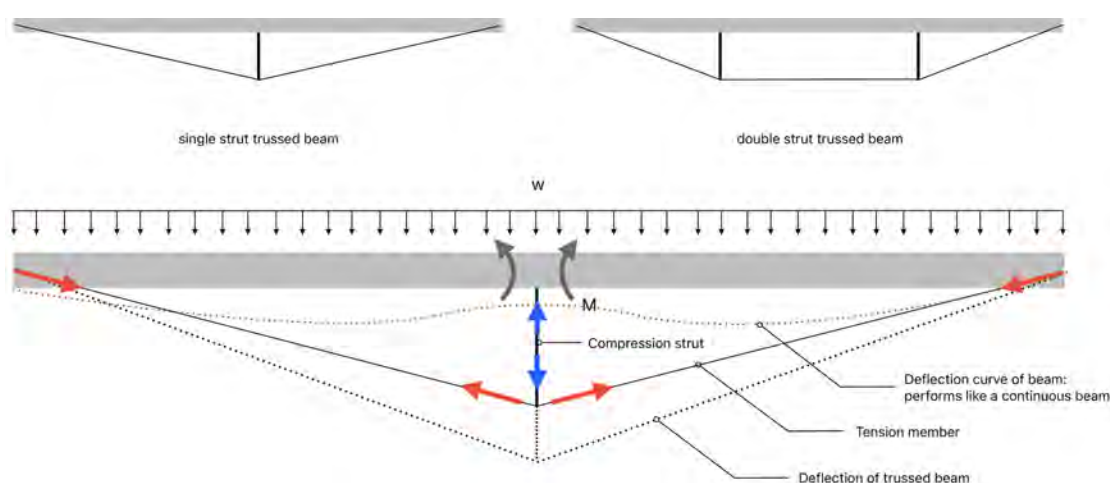


Figure E5.6: Trussed beams. Analysis of behavior.

The length of the compression strut and the angle of the cables affect the forces in the beam. More depth (longer strut and cables at a steeper angle) reduces the magnitude of forces in the cables and beam.

The continuous upper chord member of the trussed beam can be formed of one or more pieces of wood, steel or other material that can resist bending stress. The tension chords below are usually cables, rods or angles. The compression strut is typically a steel or cast steel member.

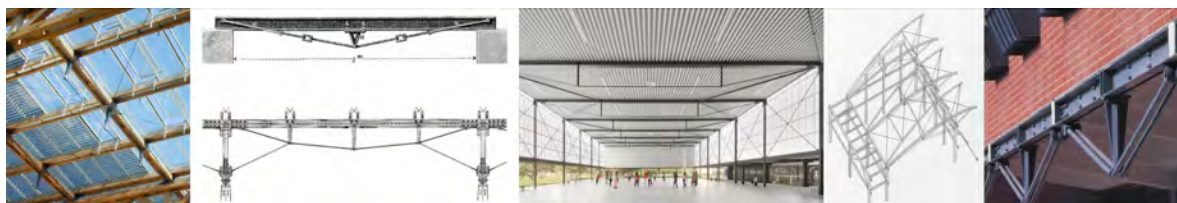


Figure E5.7: Trussed beams. Examples.



## 5.7 Composite Sections

Beams with composite sections, meaning composed of more than one material, are not uncommon. A reinforced concrete beam is a composite section. Other combinations include steel or aluminum and wood, PVC and fiber reinforcement (carbon filament), and carbon fiber reinforced plastic and polymer foam. In general, the strategy of combining materials with different properties is to take advantage of each material's capabilities, for example, combining strength and reduced weight. A strong material in tension and compression such as steel, is placed at the extremities of the beam section, held together by a web or core of a lighter but less strong material.

### REVIEW QUESTIONS

- Comment on the structural efficiency of a universal steel beam.
- In typical beams (not cantilevers) where do the maximum positive moments occur? Where do the maximum negative moments occur?
- Explain why continuous beams are more efficient than simple span beams.
- What is a composite beam? Give an example.
- Sketch the deflection curves of beams with various end support conditions. Which have the most deflection? The least?
- What are the main differences between a truss and a trussed beam?

## 6.0 Slabs, Flat Plates and Grid Frames

Flat, horizontal spanning structures that are loaded perpendicular to their surface. They include all *monolithic* slabs and flat plates as well as *reticulated* flat horizontal structures such as beam grid frames, truss frames and space frames.

### 6.1 Slabs and flat plates

A slab is a planar spanning structure whose length and width dimensions are much greater than its thickness. Unlike other surface or plate structures, it is designed to resist loads acting perpendicular to its planar surface which induce bending and curvature. Slabs are supported in various ways: walls, piers, columns and beams. They are probably the most common structural span element in use throughout the world today.

A one-way concrete floor slab is basically a wide, flat beam that bends in the direction of the span. Internal bending moments develop to resist curvature in one direction. The action is similar to a beam: for positive concave curvature maximum compression force occurs at the top edge of the slab section, diminishing to zero at the neutral plane, and then increasing to a maximum tension force at the bottom of the slab.

In general, it is the length and width proportions of the configuration of a slabs support that determines whether bending is in one direction only (one-way) or in two directions that are perpendicular (two-way). A slab spanning between two lines of continuous and parallel support, such as a two walls or beams, will be one-way regardless of the proportions. However, if support is provided on three or more edges of the slab, or by columns only, the proportions will determine the behavior. Proportions in the range from square (1:1) up to a rectangle of 1:1.5 will result in two-way bending. In longer and thinner rectangles, curvature in the short span direction will dominate and bending in the transverse direction will be minimal.

Introducing openings in a flat slab is problematic but sometimes necessary. These openings should be carefully positioned so as not overlap the beam strips between column supports. A *breakout panel* is sometimes indicated on the slab as a safe place to introduce an opening, such as for an interconnecting stair between floors.

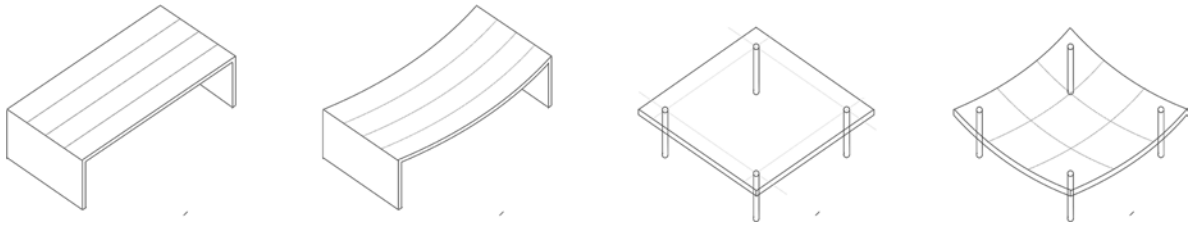


Figure E6.1: Slab bending: one-way versus two-way.

Slabs without beams integral with the edges are referred to as *flat plates*. Most slabs incorporate beams on two or more edges as they provide continuous support for better transfer of the loads as well as stiffening the slab to reduce bending in the mid-span, and hence, deflection. In r/c construction edge beams are monolithic or continuous with the slab thereby effecting a fixed moment-resisting connection. This results in *negative bending* in the slab at or adjacent to the beams and *positive bending* in the mid-span area.

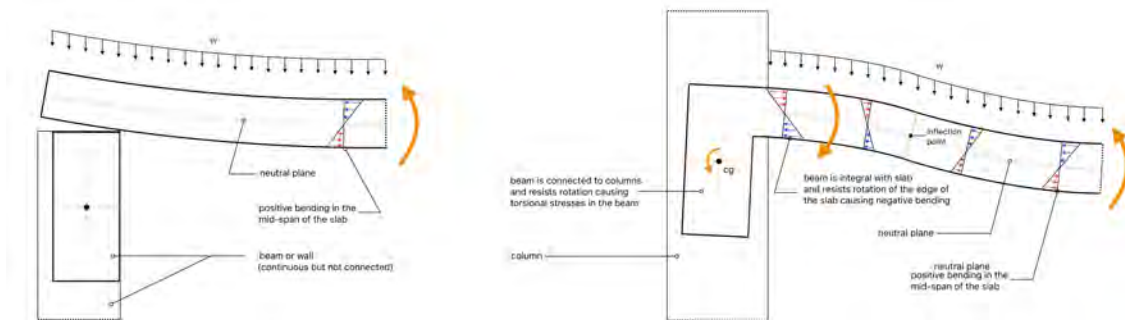


Figure E6.2: Flat plate bending versus slabs with edge beams.

For longer spans and/or heavier loading it is more economical to introduce beams in the span direction of the slab rather than increase the thickness of the slab. Reusable pan-form construction techniques allow one-way *ribbed slabs* to be constructed efficiently. The rib is a thin beam integral with the slab and thereby creates the effect of a series of T-beams placed side by side. Since the slab is continuous it also spans between and over the ribs in the transverse direction to the main span, behaving as a continuous plate over several supports.

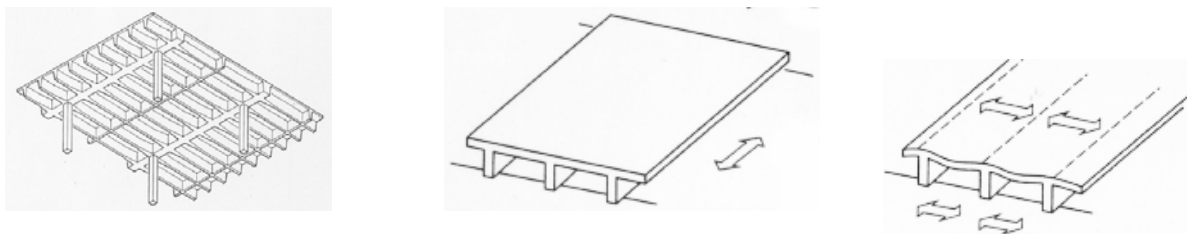


Figure E6.3: One-way ribbed slab. *Structures*, Schodek/Bechthold. Figure 10.8 p358

Again using pan-form construction techniques, ribs in both directions create a two-way slab also known as a *waffle slab*. A distinctive feature of the waffle slab is that in the

vicinity of the column, some of the pan created voids can be eliminated or “filled in” thus strengthening the shear capacity of the slab and preventing *punching shear* failure.

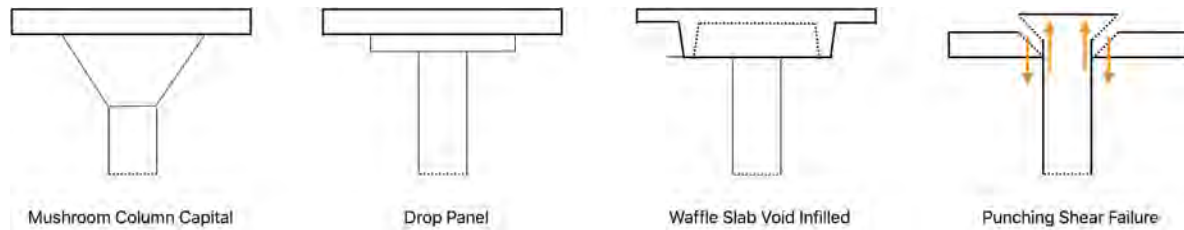


Figure E6.4: Prevention of punching shear failure.

## 6.2 Grid Frames

Grid framing is an alternative to typical parallel beam framing. A *grid* is a pattern of planar spanning members (beams, trusses) arranged at right angles to one another. The grid can be orthogonal with the supporting structure or arranged at a  $45^\circ$  angle, which is referred to as a *skew grid* or *diagrid*. Grid frames behave in a manner similar to a beamless flat plate. They bend in two directions provided supports are arranged in a relatively square proportion (1:1 to about 1:1.5). Load transmission to the supports is dependent on the position and direction of members relative to the loads. A lattice grid is an alternative name used when the members are trusses. A waffle slab can be thought of as a type of grid frame. Grids are less economical than conventional framing. They are more applicable to large spans where the economies of scale balance the extra cost of constructional complexity.

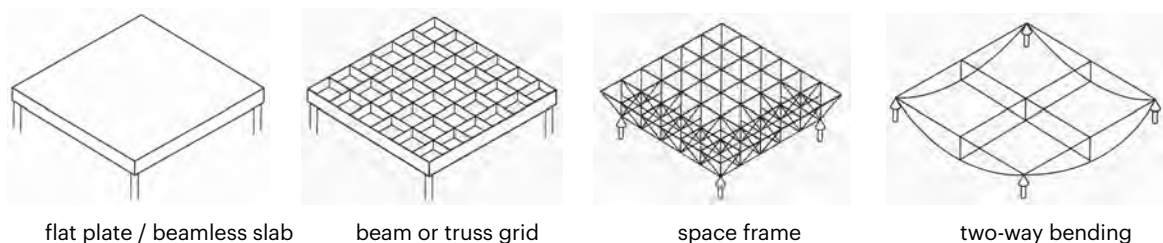


Figure E6.5: Two-way plate structures. *Structures*, Schodek/Bechthold. Figure 10.1 p353

## 6.3 Space Frames

*Space frames* are three-dimensional lattice structures that are very efficient for large spans (20-35m) with a depth to span ratio (1/15 - 1/20). Load, span and edge support conditions all play a factor in the form and required depth of the space frame. A range of configuration types based on the geometries of the pyramid (square) and the tetrahedron (triangular) are possible and include: square on square, square on diagonal, diagonal on square, square on square offset, octagon on triangle, etc.

The strength and structural rigidity obtained by three dimensional triangulated framing improves its efficiency over non-triangulated grid frames and has led to its application to a wide range of shapes and configurations (e.g., Cultural Center in Azerbaijan by Hadid). As with planar space frames, the main challenge is in the design of the joints. Proprietary systems of space frames have emerged based primarily on the joining elements. The Mero KK space truss system was the first commercially available and most widely used system to date.

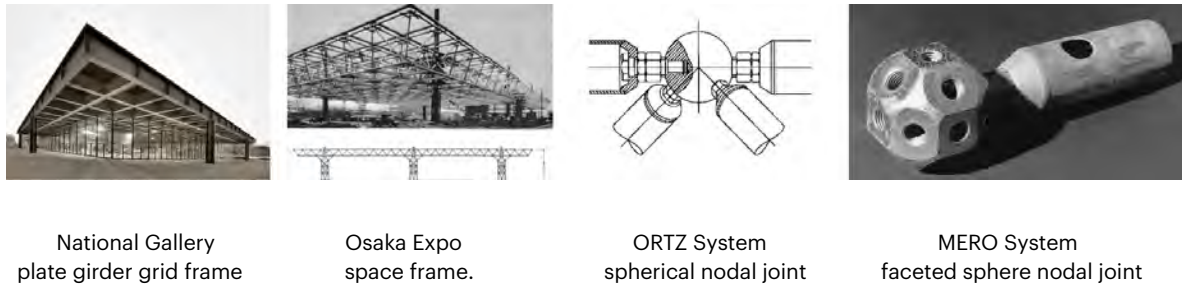


Figure E6.6: Grid frame and Space frame

As the span increases, the depth of a space frame must also increase. This leads to longer and thicker members (especially compression members that must resist buckling), and heavier joint nodes. Some designs introduce a horizontal third layer between the top and bottom chords primarily to brace the compression struts.

A particular issue for space frames is the transfer of load to the vertical support structure (column or wall). Since these structures are long-span they transmit very large reaction forces to the vertical support. If columns are used, the top of the column can use an intermediate branching structure that will spread the column support to four frame nodes rather than one. Grid frames of beams or trusses tend to have heavier members and can safely be supported at a single point.

It is good practice, if possible, to position the columns supporting a space frame inside the perimeter of the structure, allowing a cantilever on all sides of  $\frac{1}{4}$  to  $\frac{1}{3}$  of the span. This will reduce the mid-span moment by distributing a portion of it to the bending moment of the frame over the supports.

## REVIEW QUESTIONS

- What design situation might require a two-way waffle slab?
- For the same span and loading, which type of slab would be the thinnest? One-way beam-less slab, two-way flat plate or waffle slab?
- What benefits does the edge beam on a slab provide?
- What is the main difference between a *grid frame* and a *space frame*?
- Explain what “punching shear” is and how a drop panel can prevent it.

## 7.0 Structural Panels, Folded Plates and Shells

Span structures referred to as *surface active* include structural panels (wall girders as opposed to flat plates), folded plate structures, and shells. These elements all have a small dimension of thickness in comparison to length and width. They are most efficient when loaded parallel to their surface and develop in-plane stresses.

### 7.1 Structural Panels

Structural panels (deep beams) can span between two or more supports and support loading on their edges and parallel to their surface. Because of their large depth relative to span ( $\text{span/depth} < 2$ ), their behavior is different from that of beams, transmitting loads more through in-plane tension and compression rather than bending. For this reason *shear forces* tend to dominate over bending moments. Deflections accordingly are relatively small.



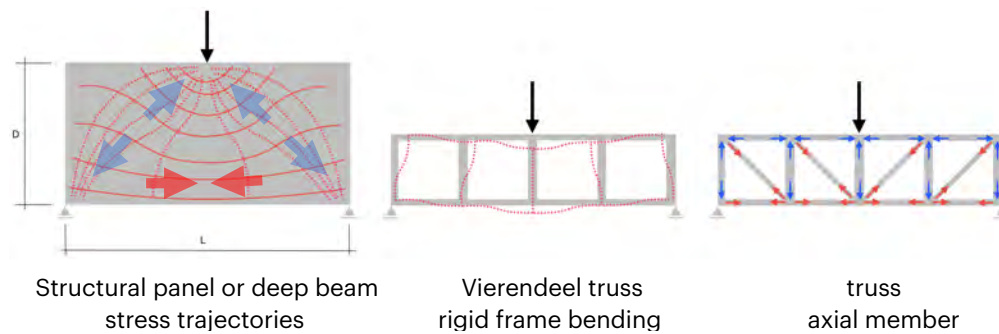


Figure E7.1: Comparison of the behavior of panel, frame (Vierendeel truss) and truss.

## 7.2 Folded Plates

Plate elements that join together on their edges and form extruded shapes with a deep section can span long distances efficiently. The individual plates themselves are thin relative to their length and breadth, and therefore prone to buckling. They are also weak in bending with loads applied perpendicular to their surface. By joining along edges, plates brace each other and transfer loads through in-plane forces. As a combined unit, however, folded plate structures are essentially deep, open-section beams with some plates developing compressive bending stress and others, tensile bending stress, or both. Folded plate structures have been made of cast (in situ) concrete, pre-cast concrete, steel, aluminum and wood.

In addition to longitudinal beam action, the bracing provided by the adjacent plate edges provides continuous vertical support to the edges of the supported plate, effectively causing the plate to behave as a one-way slab in the *transverse* direction. In multiple adjoined structures with rigid connections, the transverse one-way slab strip behaves like a continuous beam supported on several supports.

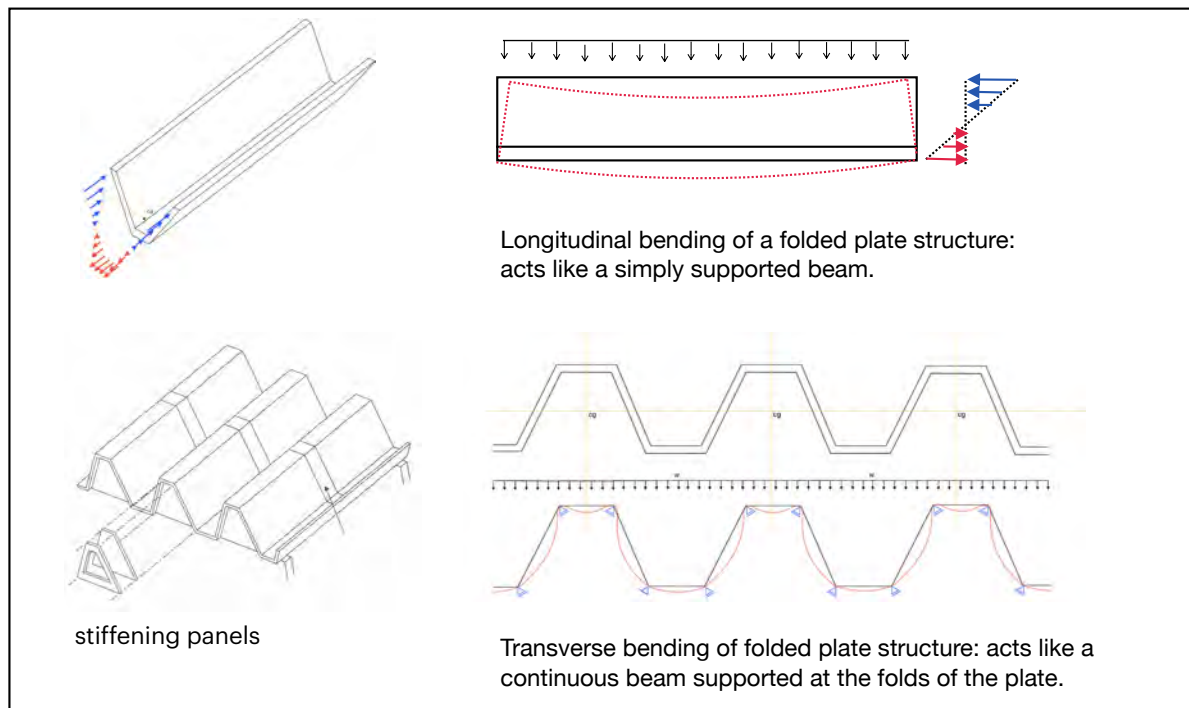


Figure E7.2: Behavior of folded plate structure.

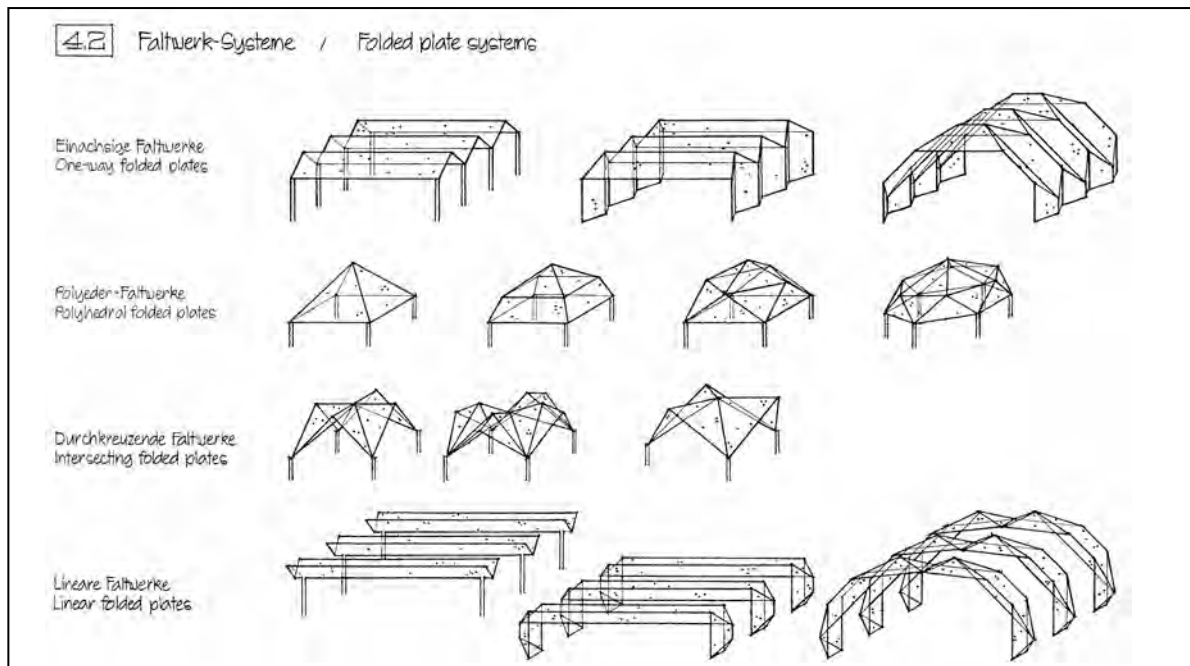


Figure E7.3: Folded Plate Structures. Various Forms. *Structure Systems*, Heino Engel. p214

The plate elements that combine together to form a folded plate structure can be monolithic, multi-layered or stressed skin (a sandwich panel with stiffening elements between two thin sheets of material). The US Air Force Academy Cadet Chapel (SOM-Walter Netsch 1962) is made of tubular steel tetrahedron frames with aluminum panel surfacing. It is essentially a 3D truss frame in the form of a folded surface (structurally not a folded plate structure).



Figure E7.4: Folded Plate Structures. Examples.

### 7.3 Shells

Shells are surface active structures with curvature. Formally there are three types of shells:

- *single curvature shells*. The surface is composed of an arc and straight lines. An examples are barrel shells, cylinders and cones.
- *dome shells*. Domes (hemispheres) have curvature in two directions (latitude and meridian) but the curvature is all *convex*. Domes can be truncated or sliced by planes to create non-hemispherical shapes. They are synclastic.
- *saddle shells*: These shells have surfaces with both positive (convex) and negative (concave). They are anti-clastic. They are generated from parabolas, concave in one direction and convex in the opposite direction. The most common is a hyperbolic paraboloid.

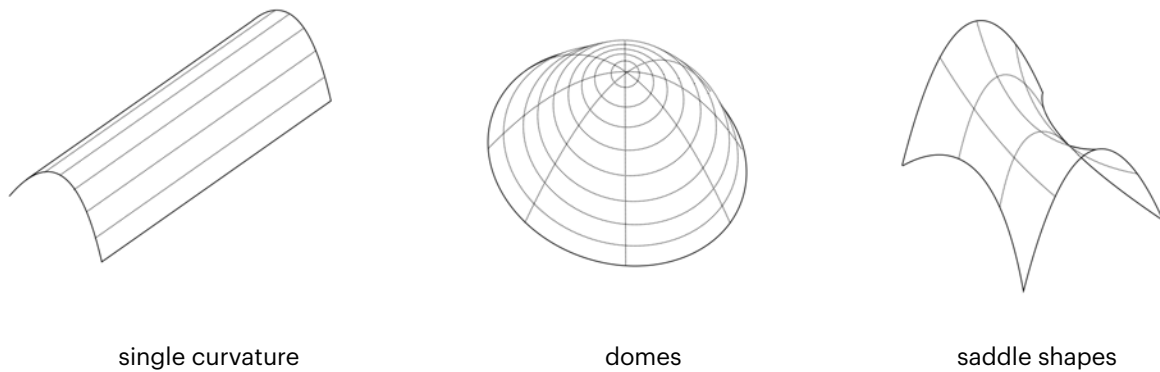


Figure E7.5: Three types of shell surfaces.

Surfaces of regular geometry are generated in several ways:

- *rotational surfaces* are generated by a curve rotating about an axis (can be a semi-circle, ellipse, parabola, etc.)
- *translational surfaces* are generated by sliding a plane curve over another plane curve (cylindrical and elliptical paraboloids)
- *ruled surfaces* are generated by sliding the end points of two lines along two individual plane curves (conoids and hyperbolic paraboloids)

Complex surface forms have also been described mathematically using analytical parametric expressions. Furthermore, digital applications employing lofted surfaces, spline curves, etc. enable designers to generate complex free-form surfaces.

Not all curved surfaces are what would be considered structurally efficient. It is necessary that the surface has a shape that enables membrane (in-plane) forces to develop while minimizing bending stress. The most efficient surfaces are those that are *funicular* in form. Various modeling techniques in the past were devised to determine the exact shape (e.g., soap bubble films) of these surfaces. Today structurally efficient forms (with only tension and compression membrane forces) can be generated digitally.

The analysis of spherical shell structures (e.g., hemispherical domes) is slightly complex and cannot be seen as analogous to the behavior of independent arches rotated around a central axis (forming the rotational surface of a dome). The distributed loading on a dome (the combined dead and live load estimate) induces membrane forces both from the top of the dome to the bottom (meridional forces) and circumferentially (hoop forces). The meridional forces increase to the bottom and have an outward thrust (similar to an arch) that is contained by the use of a *tension ring* around the base of the dome. Hoop forces meanwhile vary from compression in the upper portion of the dome, to tension in the lower portion. These hoop forces in the dome (perpendicular to the meridional forces) are critical because they prevent any bending forces from developing in the shell (only membrane forces are present).

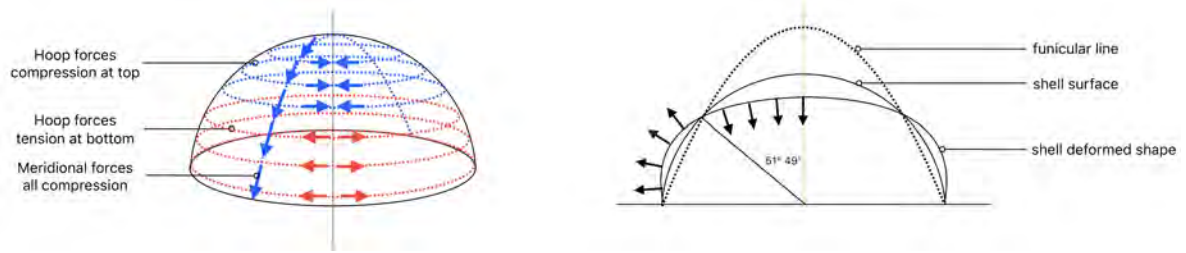


Figure E7.6: Analysis of a spherical shell.

Cylindrical shells are translational surfaces that are supported on walls or piers. If the shell is supported along its longitudinal edge by a wall or a stiff beam, the curved surface behaves structurally like a series of parallel vaults. When the shell becomes long and is supported by an edge beam without sufficient stiffness, the shell begins to perform more like a beam in the longitudinal direction. Transversely, the arch action that previously existed in the shell supported on a wall or stiff beam capable of resisting the outward thrust at the base of the arch, is no longer present. The longitudinal edge of a shell with no stiffening edge beam will actually deflect inward. Long cylindrical shells supported on columns are referred to as *barrel shells*. The addition of transverse stiffeners at the ends or at points along its length increases the load carrying capacity of the barrel shell and it performs more like a beam, especially when the length is 3x or more the width of the shell.

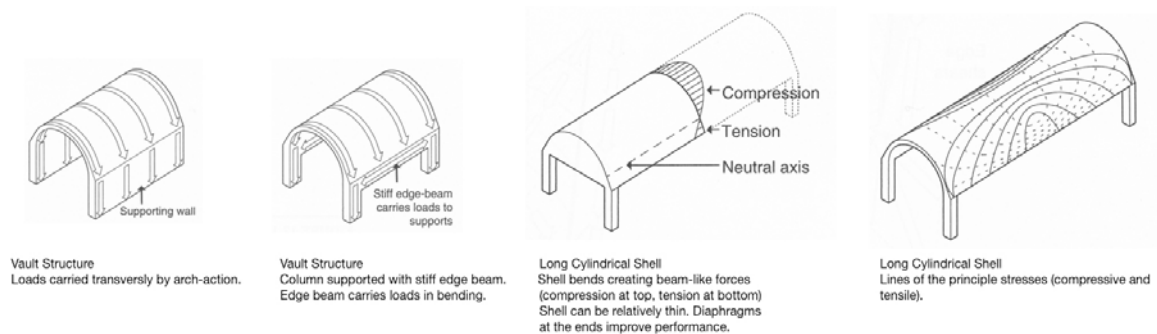


Figure E7.7: Cylindrical Shells. *Structures*, Schodek/Bechthold. Figure 12.14 p413

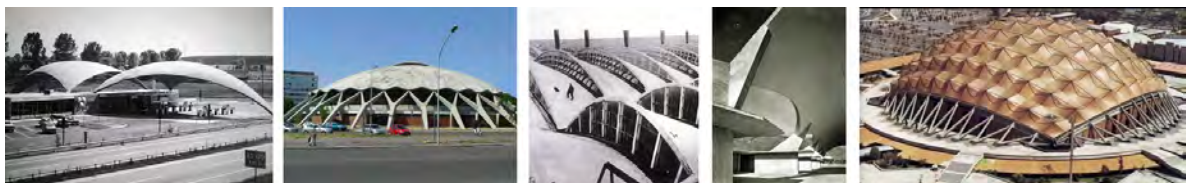
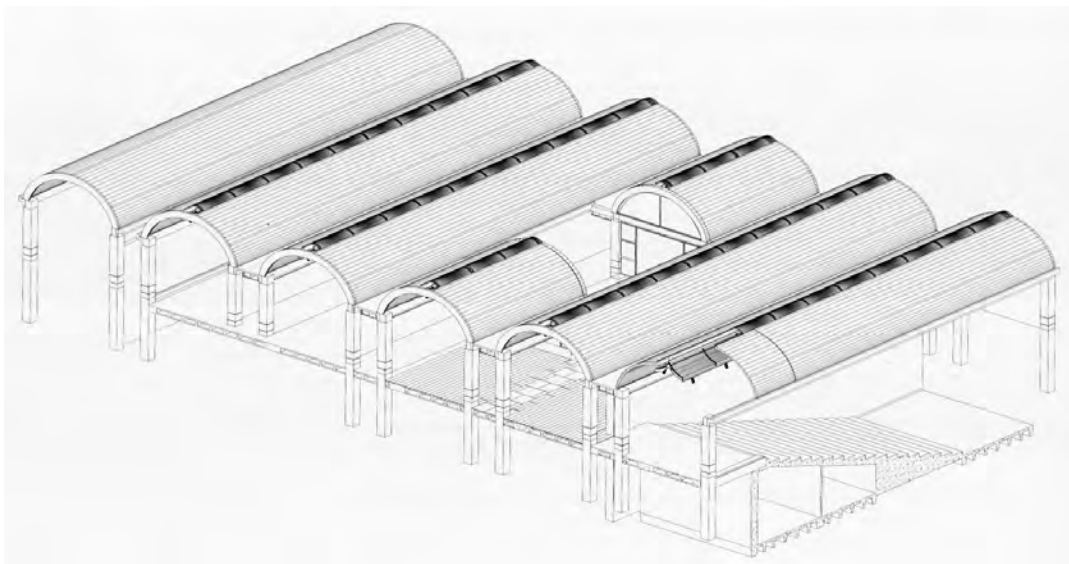
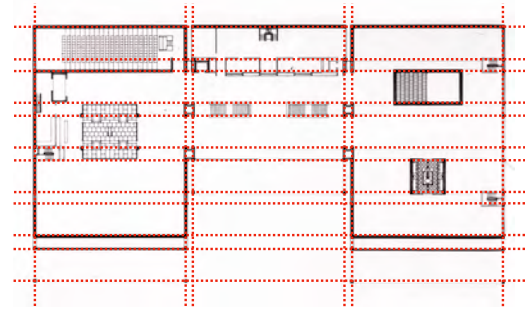


Figure E7.8: Shell Structures. Examples.



### Case Study: Kimbell Art Museum (1972)

The Kimball Art Museum in Fort Worth, Texas, was one of Louis Kahn's major works and most admired buildings. It was also one of the most successful collaborations with his long time structural engineer consultant, August Komendant. The design of the Kimball was focused on the introduction of natural light into the art galleries and no element was more critical than the roof. Kahn explored many sectional possibilities and finally arrived at a scheme featuring a 30m long barrel vault supported on four columns with a narrow skylite slit along the length of the crown. These shells are used over the entire plan and provide a repetitive formal structure to the museum.

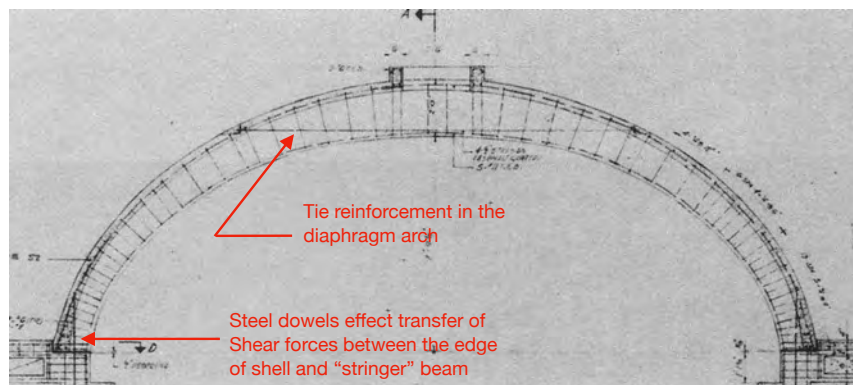
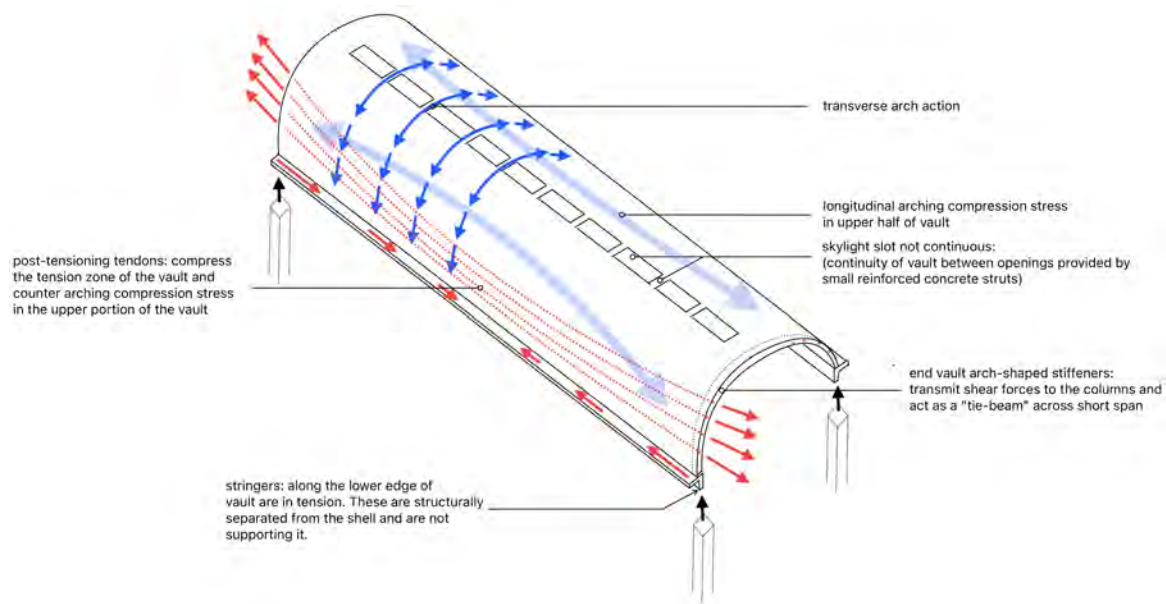


Cutaway isometric drawing of the Kimball (*Louis I Kahn*, Robert McCarter. 2005. p365)

## Structural Analysis of the Vaults

The roof vaults of the Kimbell are *cylindrical shells* supported on columns at the four corners. The introduction of a skylight slit at the crown of the shell creates some controversy as to whether the structures behave as shells acting as beams in the longitudinal direction and using arch-action to collect and transmit loads transversely, or, is the shell essentially two long, curved posttensioned beams (the slit effectively cutting the shell in half) prevented from overturning by the bracing of the concrete *struts* between the ten skylight openings?

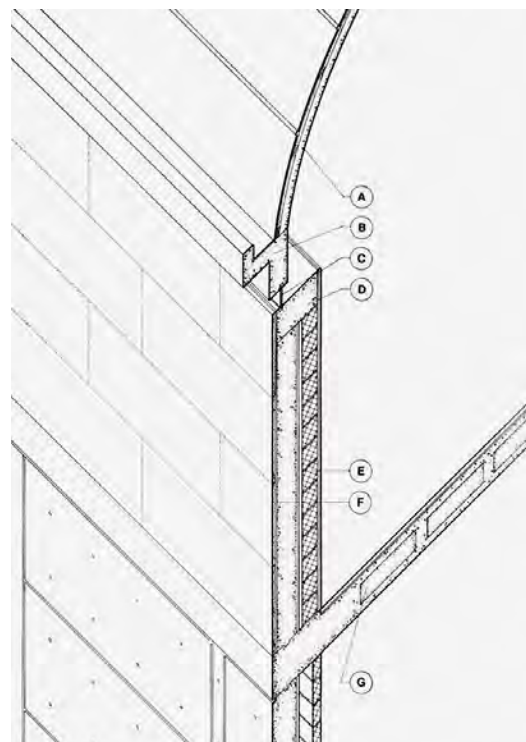
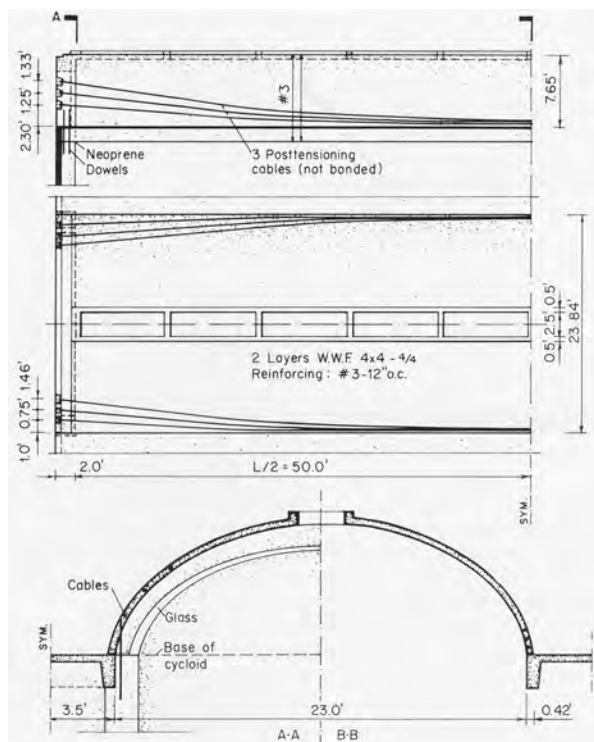
Architect, structural professor and author (see Aurelio Muttoni, "An Analysis of the Structure" in *Louis I. Kahn: The Construction of the Kimbell Art Museum*, 1999 by Skira editore) explains the behavior of the shells as the former. In the transverse (short span) direction, arch-action compressive forces are collected and transferred through the shell by longitudinal compression, which in turn is resisted by the post-tensioning strands. At each end of the shell there is an arch diaphragm (thicker at the crown and thin at the bottom) that stiffens the shell and also absorbs outward directed forces near the crown acting as a tie. The beams below the longitudinal edges of the shell (stringers) absorb shear stresses (generated by the longitudinal arch compressive stress) and are in a state of pure tension. They do not support the shell or resist the arching compressive stress as they would in a conventional *barrel vault*. The shell instead behaves more like a very stiff beam.



## Structural Analysis of the Vaults

The profile of the shell is called a *cycloid*. The shape is generated by the trajectory of a point on the circumference of a circle translating along a horizontal line. Its proportions of height and width are related by  $\pi$ . In the earlier schemes Kahn had proposed semi-circular vaults that, for a bay width of 6.71m would be 3.35m height; much taller than the height of 2.30m of the cycloid. The desire to keep the height of the building low was partly responsible for choosing the cycloid shell profile.

The arch diaphragms in the ends of the shell, required for stiffening, were originally proposed as constant depth (approx 25cm). Komendant suggested that they be shaped in the form of a two-hinged arch, deep at the crown and narrow at the ends, to reflect their actual structural performance. Kahn accepted this revision and further altered the width of the glass strip that visually separates the shell from the non-supporting wall below, making it thin at the top (4in) and wider at the bottom (9in). This has the effect of causing the infill wall profile to match the cycloidal profile of the shell.

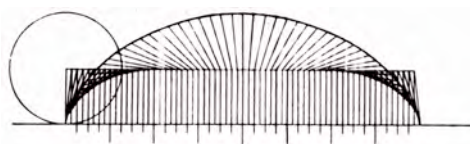


Above: Details of the structure of the shell. Note the position of the pre-stressing cables.

Below left: interior corner detail. Below right: interior elevation with channel-type marginal beam accommodating the air ducts.

Below far right: generation of cycloidal curve.

Wall section. A\_4in r/c, 2in rigid insulation, 2 layers of 3/8in ply, seamless lead sheeting. B\_"stringer" beam/gutter. C\_9in glass strip. D\_r/c beam. E\_8in block wall with 2in rigid insulation. F\_concrete wall with 7/8in travertine marble facing. G\_concrete slab with foam block inserts. (*Details of Modern Architecture 1928 to 1988 Volume 2*, Edward R. Ford)





The hyperbolic paraboloid shell (hp-shell) was extensively explored and developed as a roof structure by the architect Felix Candela (1910-1997). Hp-shells are capable of moderate spans with relatively small thickness (<50mm). They can be shaped and combined into an unlimited number of forms. Because of their particular geometric configuration, the formwork for casting is relatively simple and prestressing is applicable.

The hp-shell is generated in one of three ways: a) the rotation of a hyperbola about an axis b) a parabola translating along another parabola such that the plane of the moving parabola remains parallel (parabola A'-C' moving along parabola B-D), or c) the ends two straight lines translating along two straight lines in parallel planes but having opposite slope (this is also a type of *ruled surface* and is a portion of the hp-shell in figure b below).

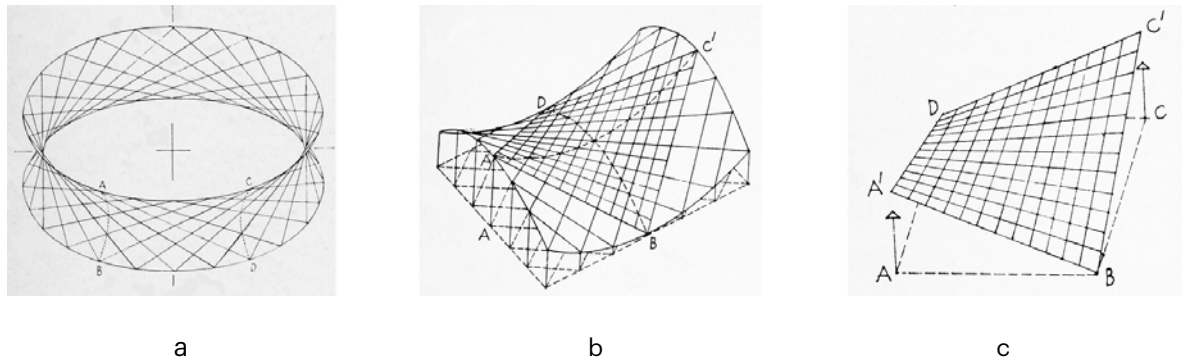


Figure E7.9: Generation of HP-Shells. *Surface Structures in Building*, Angerer. Figures 82-85 p55

The behavior of a hp-shell structure is suggested by the opposing curvatures of the shell: convex parabolic lines (arching upward) can be envisioned as compression force (arch-action) trajectories while the perpendicular concave parabolas are tension force (cable-action) trajectories. If the curvature is too little and the surfaces become flat, then bending forces will dominate and the shell behavior will be more plate-action.

At the edges of the hp-shell the resultants of the tension and compression forces are concentrated. These edge shear forces transfer much of the load of the shell to the abutments or vertical support elements.

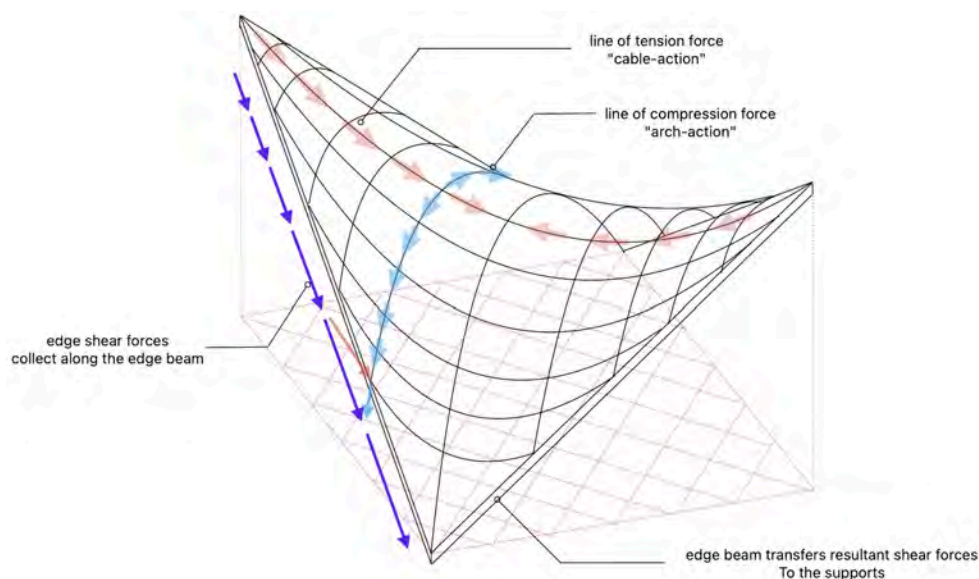


Figure E7.10: Behavior of HP-Shells.



Since a hyperbolic paraboloid surface can be generated with straight lines (see type c above) the formwork to create the shell is much simplified and different combinations of individual shells is possible.

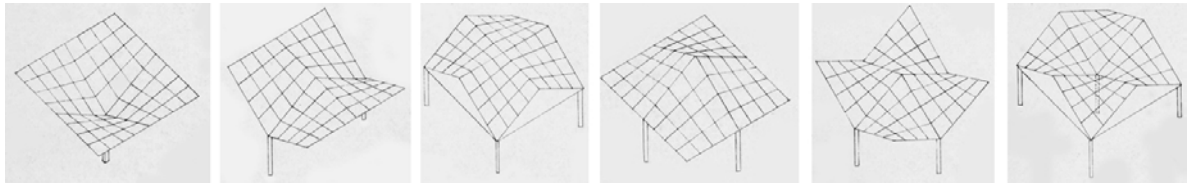


Figure E7.11: Combinations of flat HP-Shells. *Surface Structures in Building*, Angerer. Fig. 87 p57



Figure E7.12: HP-Shells. Examples.

## REVIEW QUESTIONS

- How does the structural behavior of a horizontal plate (e.g., a slab) differ from that of a vertical plate (e.g., a deep structural wall panel)?
- What force dominates in a wall plate loaded on edge and spanning between two points?
- What element is needed to make a folded plate structure stiff?
- In a long barrel shell (e.g., Kimbell art Museum) which forces are dominate? Transverse arch-action or longitudinal bending?
- Describe the membrane forces in a hemispherical shell structure. What are the meridional forces? Compressive or tensile?
- How must a cylindrical shell be supported for it to have arch-action like a vault?
- Describe the structural behavior of a hyperbolic paraboloid shell in simple terms.

## SELECTED REFERENCE

- 1) *Structures*, Daniel L. Schodek and Martin Bechthold, 2014 7<sup>th</sup> Ed., Pearson. (Pt. II Ch. 4-6, 8-12)
- 2) *Structure and Architecture*, Angus J. Macdonald, 2001 2nd Ed. (Ch.4]
- 3) *Structure in Architecture*, Mario Salvadori and Robert Heller, 1963, Prentice-Hall.

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## Section F Vertical Building Structures

### INTRODUCTION

A *vertical building structure* is a support system for a building with multiple floor levels above ground. Buildings may be classified as i) low rise ii) middle or mid-rise and iii) high-rise. The selection and economical design of the structural system will vary depending on the height of the building.

### TOPICS

#### 1.0 Low and Mid-rise Building Structure

The structure of low and mid-rise buildings is typically a conventional frame (RC, steel or glu-lam wood or timber wood framing) or standard wall construction (RC, masonry or cross laminated wood panel). Lateral stability and resistance to lateral forces must be ensured but is generally not a primary structural consideration as it would be in hi-rise.

#### 1.1 Types of Structural Systems

- *Wall systems* provide both vertical structural support and enclosure. Openings in walls are either holes or subtractions of portions of the wall. The material that walls have been constructed from (in historical order) are: 1) stone masonry 2) clay brick 3) wood (solid timber and then later, light stud wall framing) 4) monolithic r/c 5) concrete masonry 6) engineered wood panel construction (CLT). Recently there have been attempts to revive traditional forms of straw bale and compacted soil construction methods for reasons of sustainable design.

*Fenestration* in wall systems consist of openings as either “holes” or subtraction of the wall. Continuity of load support above the openings requires some form of structural span. The support over a wall opening is referred to as a *lintel*. Only r/c wall construction requires no lintel as the concrete spanning above the opening effectively acts as a beam.

Wall systems are rigid and resist lateral forces parallel to the wall plane. *Parallel wall* structures must be designed so that the wall thickness-height ratio is sufficient to resist lateral forces. Otherwise buttress piers or a diaphragm floor or roof may be used for resisting perpendicular forces.

- *Frame systems* are the most common form of structural support. Frames consist of vertical support columns and some form of horizontal span element, typically a beam. Almost all framing systems are made of steel, concrete, or wood. Combinations of two of these are possible but uncommon.

Steel and wood frames are generally considered as *pin connected assemblies*. The connections between the columns and beams in a pin connected or *hinged* frame are not fully rigid and allow for *some* rotation of the end of the member. This means that bending moments are very small or zero at the ends of members. It also results in stability problems of side sway that require some form of resisting structure. A rigid lift core is often adequate to resist all the lateral force acting on the building. The walls of the core act as *shear walls* remaining stiff and transferring the lateral loads into the foundations in the ground. Diagonal bracing systems also offer lateral resistance. *Cross bracing* with flexible cables is a form of diagonal bracing.

Reinforced concrete frames generally don't require additional lateral restraint. In situ, poured-in-place concrete frames have continuous and reinforced rigid connections between beams and columns that stiffen the frame. Bending forces develop not only in the span but also at the joints. Steel frames can also be designed with rigid and semi-rigid beam-column connections that offer lateral restraint. This is usually accomplished with welded connections as opposed to bolted assemblies.

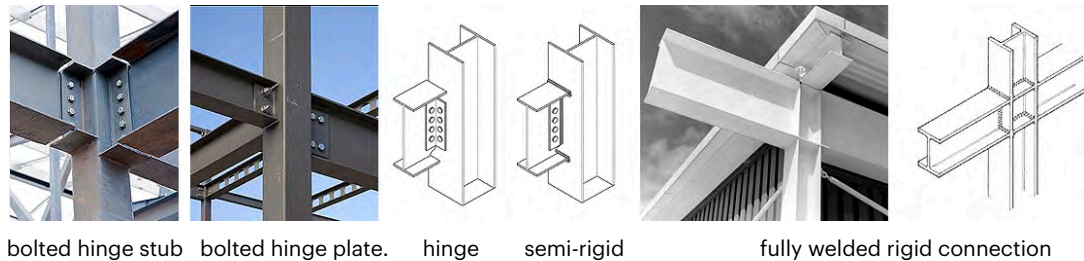


Figure F1.1 Steel Framing Connection Types.

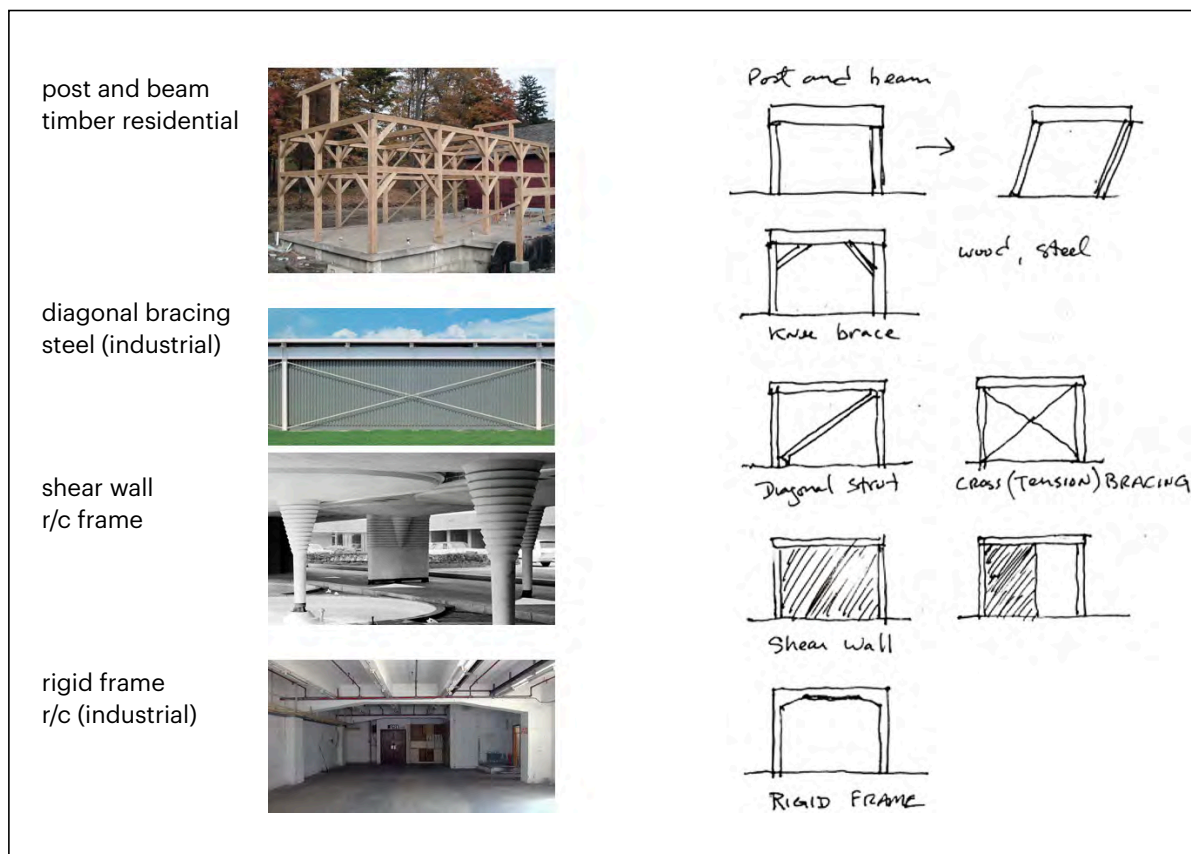


Figure F1.2 Types of Frames and Lateral Bracing.

## 1.2 Structural framing (floors)

- Steel decking supported on steel beams. Decking may be form deck (no contribution to span support and used as formwork for the concrete structural slab) or composite decking (designed to bond with the concrete slab and contribute to the structural support). A third type is cellular decking. Cellular deck is a type of decking with closed cells; a steel plate is welded to one side of a standard deck. The cells can accommodate acoustical insulation as well as electrical wiring. Roof deck is metal

decking designed for lighter loads. Usually roof deck supports live loads and non-structural roofing materials.

- Wood joist framing with sub-flooring. Wood joists can be supported on wood or steel beams and used to support subfloor panels of plywood or other materials. Primarily used in residential construction. Joists may be in the form of solid lumber or engineered wood products such as glu-lam, LVL or PSL. In addition, built-up sections may be used such as box beams, spaced beams, flitch beams (two solid pieces of lumber with a steel plate between) or prefabricated trusses and I-joists.
- Wood decking. Structural tongue and groove lumber that forms a structural base support for flooring. Also can be used for roof support.
- Stress-skin panels. A prefabricated assembly of plywood facing material bonded with adhesives under heat and pressure to an inner framework of lumber beams and cross bracing. Can achieve greater spans and can incorporate insulation.
- Cross laminated timber. CLT is a new structural floor framing system. It is essentially a wide plank one-way span system. Prefabricated and with excellent fire protection (2 hr). CLT panels can be 3m wide and as long as 16m max.
- Precast hollow core slabs. These prefabricated units can be supported by concrete or masonry walls, as well as steel, site cast or precast concrete frames. They can be made with lightweight concrete and can incorporate prestressing for greater strength. A 50-90mm concrete finish with steel bar or wire mesh bonds with the slab to become a composite structure.
- Reinforced concrete slabs. R/c slabs are used for both floors and roofs. There are four basic types of concrete slabs.
  - i) *Flat plate*. Simple to construct. Minimum structural depth (no beams). May require special treatment around the column head to counter punching shear. Two-way spanning unless configuration of support creates a rectangular bay proportion of  $L_1/L_2 > 1.5$ .
  - ii) *Beam slab*. A flat plate with edge beams that stiffen the slab against deflection and provide better shear transfer at the columns. Typically two-way but dependent on the configuration of support.
  - iii) *Ribbed slab*. One-way slab with closely spaced joists (thin concrete beams) created with removable metal pan formwork. Depth dependent on the depth of joists plus slab, which can be thinner than in beam slab or flat plate for similar spans. Greater load carrying capacity.
  - iv) *Waffle slab*. A two-way ribbed slab created with square metal domes (pans that create the voids) held on flat formwork. Depth can be less than ribbed slab for similar span. Column support should be as close to square proportion as possible for maximum efficiency.

### 1.3 Vertical supporting elements: columns, piers and walls

Columns are vertical supporting elements whose length is much greater than its cross sectional dimensions. Columns can be formed of any material capable of resisting compressive stresses. The most important parameter in the design of a column is its slenderness. Columns generally fail by buckling (instability) before the stress in the column reaches the maximum compression stress of the material.

- Column buckling failure was investigated by the mathematician/physicist Leonhard Euler in the 18th c. Euler found that the critical load that would cause a column to buckle can be predicted by the formula:



$$P_{cr} = \pi^2 E I / (L_{eff})^2$$

where:  $E$  is the modulus of elasticity of the material,  
 $I$  is the moment of inertia of the section, and  
 $L_{eff}$  is the effective length of the column.

This is important as it tells us that:

- a) a stiffer material ( larger  $E$  ) increases  $P_{cr}$
- b) a more efficient section, that is, a better configuration of material ( larger  $I$  ) increases  $P_{cr}$
- c) a larger effective column length (  $L_{eff}$  ) decreases  $P_{cr}$  exponentially (squared)

So the most significant parameter then is the effective length of the column. The effective length is a concept that takes into consideration the type of end support condition of the column. It is the length of deflected curve of the column between hinges and/or inflection points. Basically, hinged columns will bend (i.e., buckle) under load more easily than a fixed ended column. In fact, the ratio is 2:1. This is due to the longer effective column length.

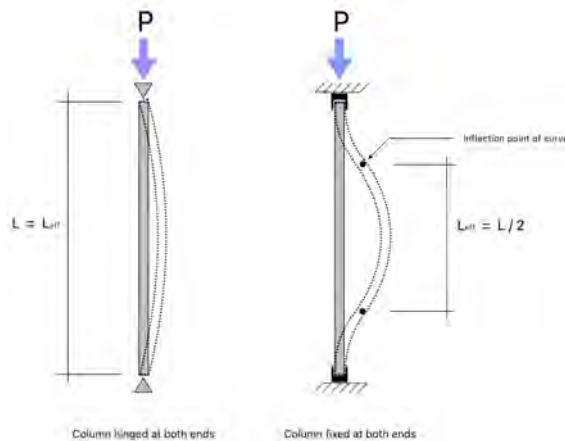


Figure F1.3 Column end support and effective column length.

Columns with a larger moment of inertia for a given cross sectional area will be more effective in resisting buckling. Since an *unbraced* column will buckle in the direction that the section is weakest in (i.e., x-x or y-y), the most efficient section for an unbraced column is a symmetric configuration in which the moment of inertia for bending about the X-X axis is equal to that about the Y-Y axis. Therefore a circular column like a steel tube will be more efficient than a rectangular tube. Note also that a circular tube will be more efficient than a square tube as it will have a slightly larger moment of inertia for the same amount of cross sectional area.

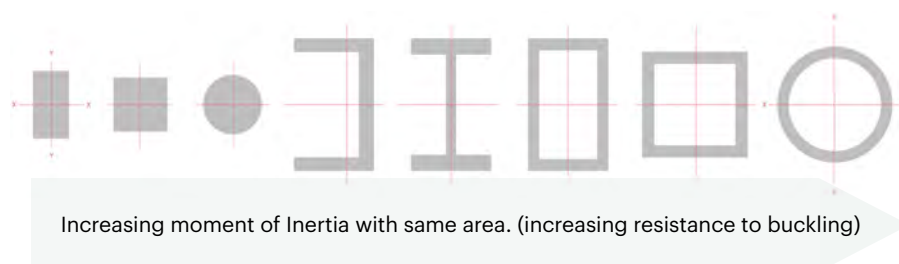


Figure F1.4 Moment of Inertia and buckling resistance (no bracing)

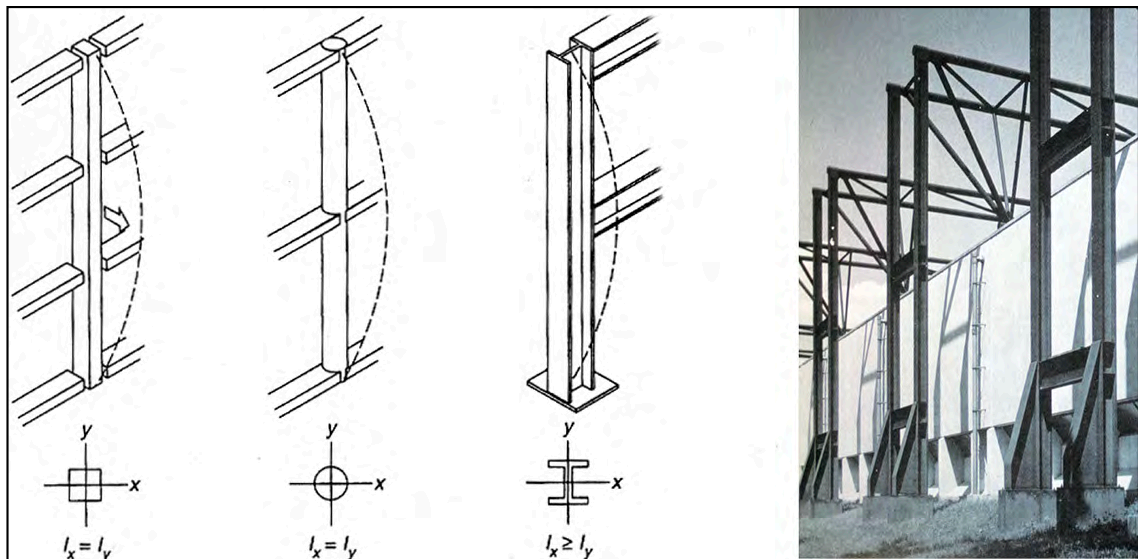
Column profile effects buckling strength. One measure of a column's resistance to buckling is its *slenderness*. Engineers use a formula that takes into account all of the parameters of the column section (area and moment of inertia) and the effective column length to evaluate the slenderness that will determine resistance to buckling.  $L/r$  is known as the *column slenderness ratio*, where  $r$  (referred to as the *radius of gyration*) is calculated as:

$$r = \sqrt{I / A}$$

A rule of thumb is that the slenderness ratio of a steel column should never exceed 200. For comparison, assuming that the column in the diagram of figure 1.3 has a solid square section, its slenderness ratio would be 97. Columns in the range of 32 - 120 are referred to as *medium columns*. Columns >120 are called *long columns*. *Short columns* are those with a slenderness ratio below 32.

A column is that is short with a slenderness ratio <32 (e.g., a solid square column of aspect ratio 1:8 has an  $L/r \sim 30$ ) is unlikely to fail due to buckling. In this case the failure would be a compression failure when the compression stress in the column reaches the maximum compressive stress of the material.

Columns may be *braced* in one or more directions and at different points in its length. A typical condition is the bracing of a column at mid-height in its weakest direction (rotation about the Y-Y axis in section). The bracing member may be a beam or other rigid horizontal element (e.g., a floor plate). The force applied to the column by the bracing member to prevent buckling is negligible.



Examples of ineffective bracing. Columns always buckle in the direction associated with the highest slenderness ratio ( $L/r$ ). The columns shown will buckle in the direction indicated. However, in the first two examples, the section is symmetrical (Square or round) and therefore there is no improvement or increase in the load carrying capacity of the column as a result of adding the bracing elements. In the third example on the right, the column is *unbraced* in its weakest direction. It will buckle in this direction and again the bracing has no advantage in increasing the buckling load  $P_{CR}$  of the column. In the photo on the right (Exeter Sports Facility by Kallman and McKinnell) the paired columns supporting the trusses use bracing members at mid-height to brace the universal columns in their weak direction.

Figure F1.5 Ineffective column bracing. *Structures*, Schodek/Bechthold. Figure 7.9 p290

Some columns use multiple shafts separated by *spacer blocks* to achieve a configuration of higher efficiency. Separating the solid shaft of a column into multiple shafts (that have a combined smaller cross sectional area than the single shaft) requires bracing along their length. Early 19c latticed columns are an example. Other strategies for bracing against column buckling are shown below.

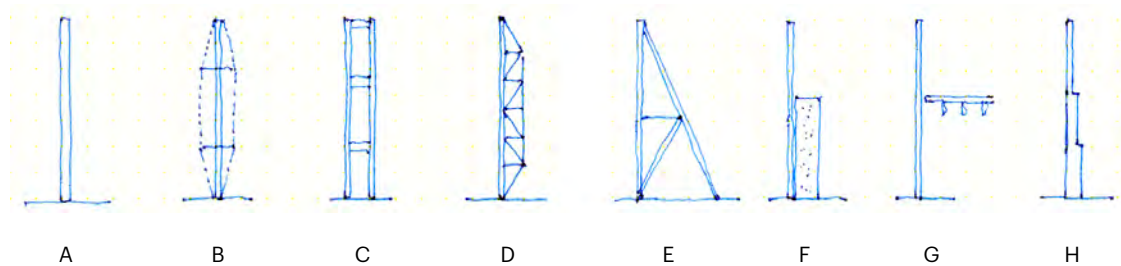


Figure F1.6 Columns. Various methods of bracing.

Since buckling failure is initiated near the mid-height of a column, increasing the thickness/moment of inertia of the section at mid-height can improve the strength of a column. This can be achieved in many ways: a *tapered* or *bowling pin* column profile is typical. Other modifications are illustrated below.

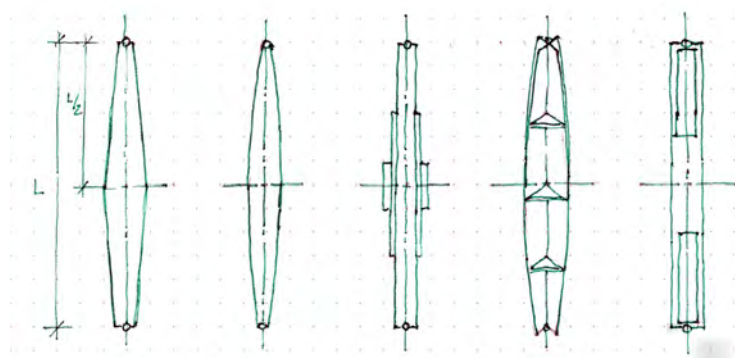


Figure F1.7 Columns. Variation in profile to improve column efficiency.



Figure F1.8 Columns. Examples.

- A rectangular column that has a ratio of sides  $>2:1$  is referred to as a *pier*. Piers and walls are subject to buckling in the direction perpendicular to the longest dimension.

## 1.4 Frame Stability

Frames must be designed to resist lateral forces such as wind. The frame uses lateral resisting structures (shear wall, diagonal bracing and rigid connections) to achieve stability in both the X and Y directions. The location of lateral resisting elements in plan is a key concern. Wind forces that strike the exterior of the building must be transferred to and resisted by these elements.

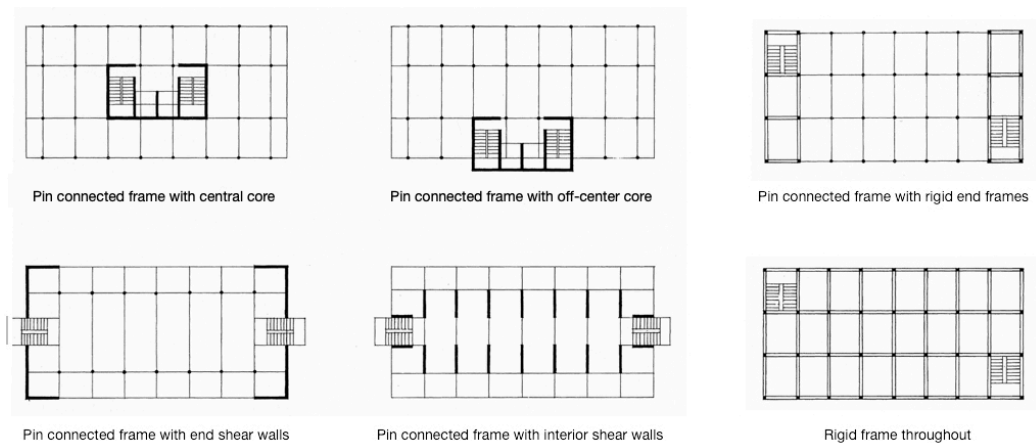


Figure F1.9 Plan configuration of lateral resisting structures.

Framing elements of the envelope are designed to carry wind forces to columns, the ground or a *horizontal diaphragm*, basically a stiff, rigid floor or roof plane through which forces are transferred by in-plane action to lateral resisting vertical structures.

## 1.5 Shear Walls and Horizontal Diaphragms

Shear walls refer to structural walls that resist lateral forces through in-plane rigidity. In most cases they also support vertical loads. Until the emergence of the hi-rise building, most structures relied on some form of shear wall to resist wind and earthquake lateral loads. A shear wall is normally made from in-situ reinforced concrete, however masonry (brick or concrete block) can be used if reinforced.

Buildings with a rectangular plan are weakest in the short span direction (more wind surface on the long side and less structure to resist it). But it is necessary to insure stability in both directions (X and Y). For example, in a steel frame with pin connections, it would be typical to use shear walls or diagonal bracing (or cross bracing) in the short, transverse span direction and rigid frames for resistance in the longitudinal direction. These might be used in conjunction with a lift core with shear wall construction.

Shear walls are located in different positions (see Figure 1.9 above) in plan and are usually discontinuous. In order for them to be effective in bracing other portions of the frame that are not designed to resist lateral force, a *horizontal diaphragm* is necessary.

Shear walls as continuous vertical wall planes must be carried to the ground. Also, to be effective, the proportions of a multi-floor shear wall should not be very narrow (like a pier, for example).



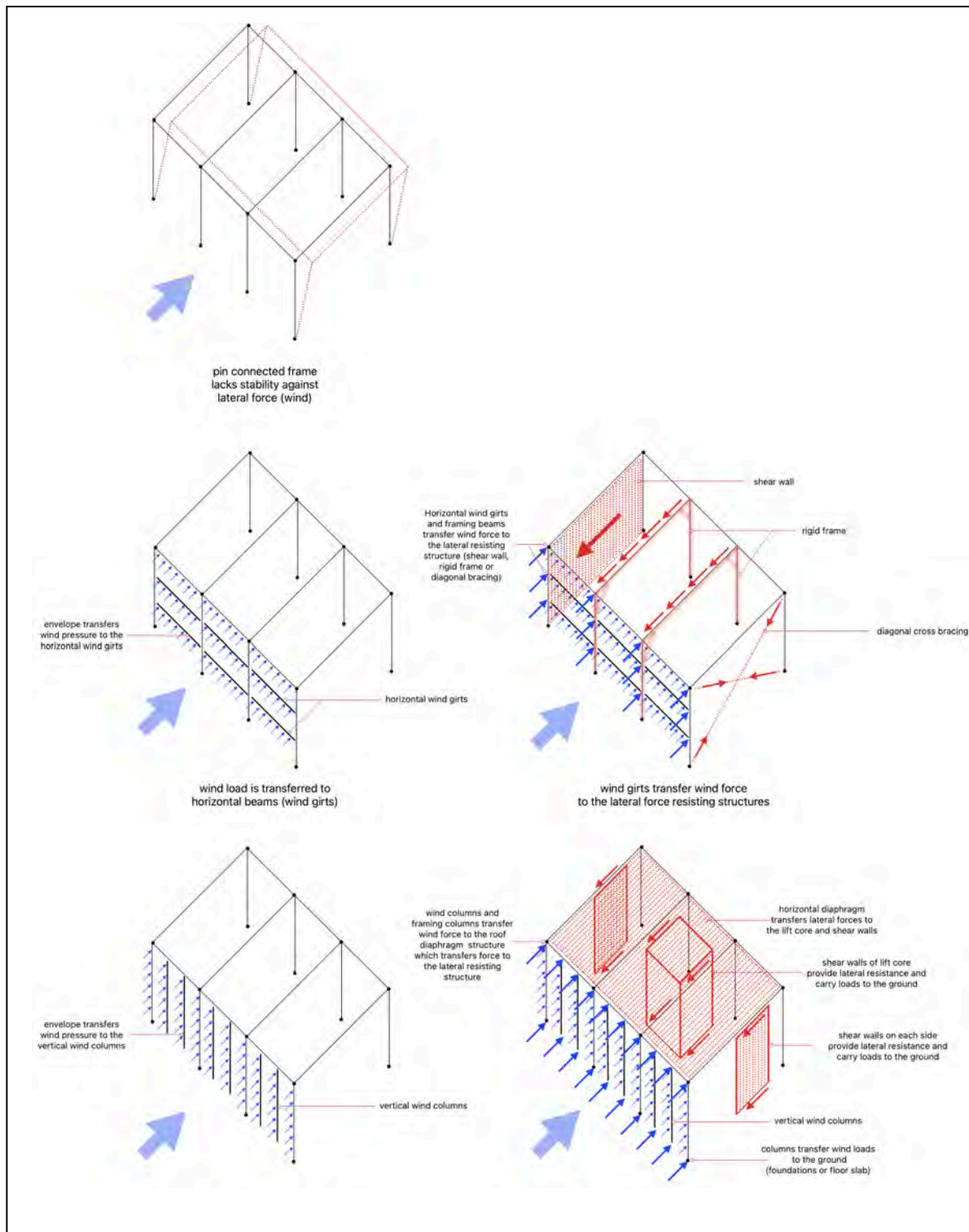


Figure F1.10 Load path of wind force to lateral force resisting structures.

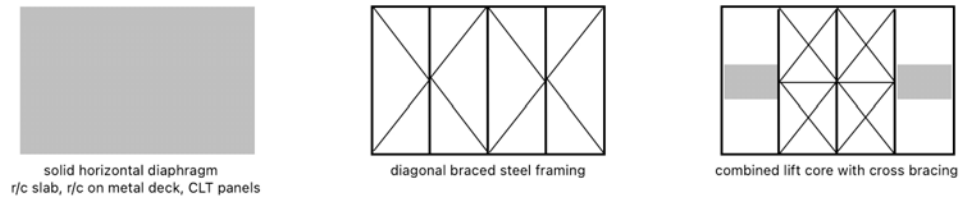


Figure F1.11 Various configurations of horizontal diaphragms.

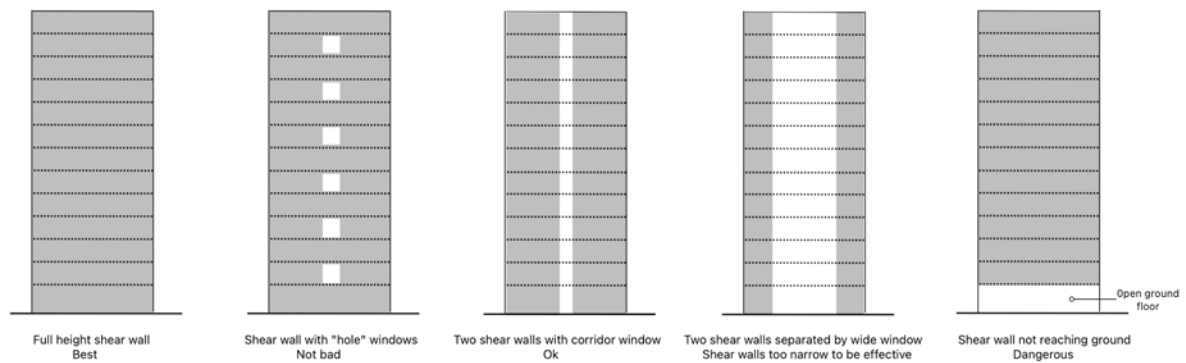


Figure F1.12 Openings in shear walls.

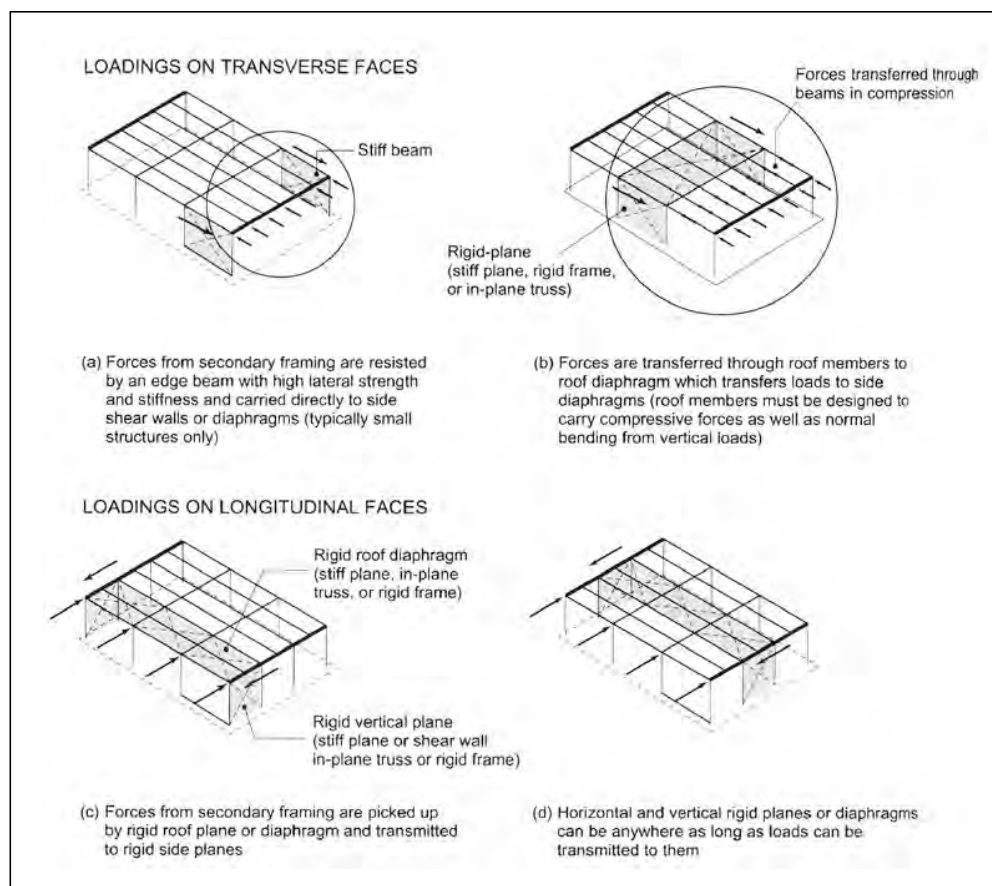


Figure F1.13 Positioning of lateral force to resisting structural planes.  
*Structures*, Schodek/Bechthold. Figure 14.4 p460

## 1.6 Vertical Transfer Structures

Functional requirements (e.g., space planning requirements) sometimes require that the spacing of columns, the column grid, change between floors. This interrupts the continuity of the column and creates a problem where a large column load of many floors must stop and rest on a beam. Typical situations where this occurs are:

- i) residential towers with a podium containing parking and/or large commercial spaces
- ii) office towers with an open public lobby at ground floor level
- iii) commercial buildings with parking in the substructure
- iv) any building that accommodates a large column free space with floors residential or office floors above
- v) a building that has to accommodate underground infrastructure such as a railway tunnel

There are several types of vertical transfer structures that are used in these cases.

- Deep beam. Concrete beam or steel plate girder. Requires additional floor height.
- Mat structure. Can be solid concrete or hollow box-type. Depth depends on the amount of load but a podium solid concrete slab can be several meters thick (2.5 - 3.5m typical).
- Full floor or multi-floor truss. Floor height remains constant but truss web members may interfere with interior circulation or exterior fenestration.
- Arch or suspension structure. In rare cases a long-span structure may be used over the entire building width carrying all the floor loads above or below.
- Deep frame (Vierendeel). A deep rigid frame can sometimes be used instead of a truss. Usually a floor deep structure, the frame has no diagonals to interfere with circulation or view.

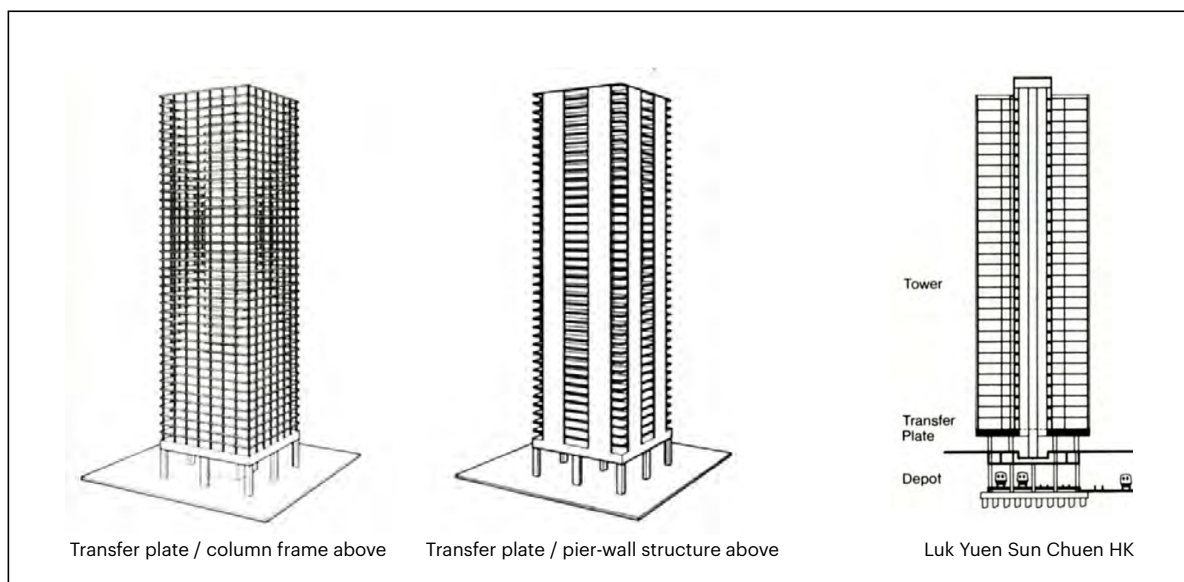


Figure 1.14 Transfer Plate. Attribution: Jack Zunz.



Figure F1.15 Transfer Plate structures. Examples.

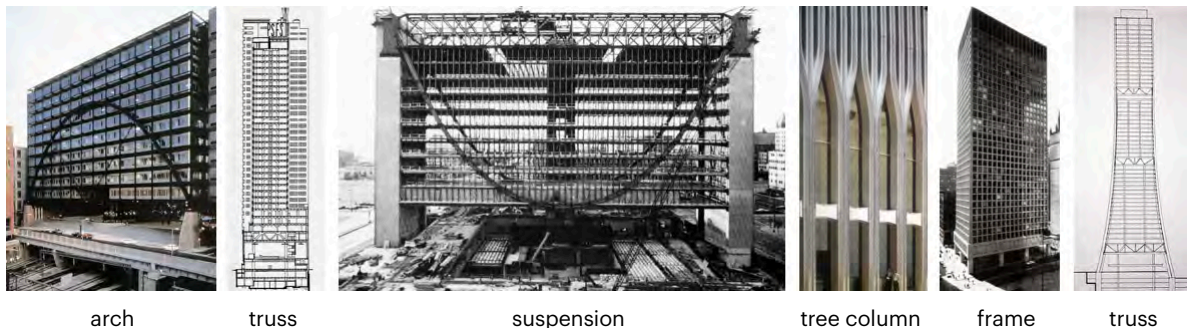


Figure F1.16 Types of Transfer Structures. Examples.

## REVIEW QUESTIONS

- What types of wall structures require lintels for openings such as windows and doors? Steel frame? R/C frame? Wood frame? Masonry frame?
- Is a flat plate concrete slab a one-way or two-way spanning structure? Explain.
- What contributes the most to column stability? Stiffer material? Better sectional shape (i.e., larger moment of inertia)? Shorter effective column length?
- A universal column (H configuration with depth equal to width) pin connected at both ends, is braced at mid-height against its flanges. The column is 6m in Length. What is the effective length?
- If a column slenderness ratio is 32 or below, what type of failure would be expected?
- Open universal sections and closed sections such as pipes and rectangular tubes are more efficient column sections. True or False.
- Describe different load paths for the transfer of wind forces to various lateral force resisting structures.
- What might be some of the problems with a shear wall that doesn't reach the ground?
- Identify *five* different structures that might be used as a column load transfer structure.

## 2.0 Lateral Forces

Wind, soil and underground water are forces which apply *pressure* to the exterior of a building. Earthquakes are also considered a lateral force as they move the ground laterally (and also up and down) and apply a horizontal shear force to the building.

### 2.1 Wind

Lateral wind force on a building can have three basic effects: overturning (catastrophic failure), bending (increased column loads, potential local buckling failure, motion disturbances), and shear racking (increased bending moments and shear in the structure and its connections, possible damage of non-structural elements).



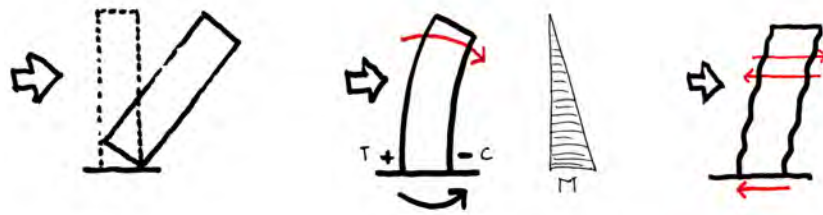


Figure F2.1 Effects of Lateral Forces on Tall Buildings

Wind is a *dynamic* force because it involves motion, but it is also a force that varies in pressure and direction over time. Predicting a *static* wind pressure on a building's surface is somewhat approximate. However, with the aid of weather measurements of wind speed and a statistical approach to probable occurrence, building codes have created wind pressure charts that enable designers to determine “worst case” wind forces on a building in any location. The calculation of static wind pressure ( $Q_z$ ) perpendicular to the surface of a building takes into account the following parameters:

- i) the maximum predicted wind speed over a 50 yr time period in a specific weather location
- ii) the height at which it is acting on a building (the effective height,  $Z_e$ )
- iii) the character of the terrain (context) of the building ( $Q_z$ )
- iv) the area of the envelope being considered (positive and negative pressures at the edges, center, and roof ( $C_p$ ))
- v) the form of the building (e.g., chamfered corners, thinness or aspect ratio, shape in plan) ( $C_f$ )

The Standard Method is the approach outlined in the *Code of Practice on Wind Effects in Hong Kong 2019*, Buildings Department of Hong Kong, for calculating the wind loads for the structural design of a building. However, for buildings greater than 200m in height or having unusual shapes or other special conditions, it is suggested that wind tunnel test investigation be made.

Wind forces on taller buildings also produce complex actions such as wind turbulence eddies and vortices. The induced motions that result from these wind forces are difficult to predict and must be determined by model simulation and wind tunnel testing. Lateral side to side motions produced by variable wind speeds perpendicular to the surface may be greater than the back and forth motion in the direction of the wind.

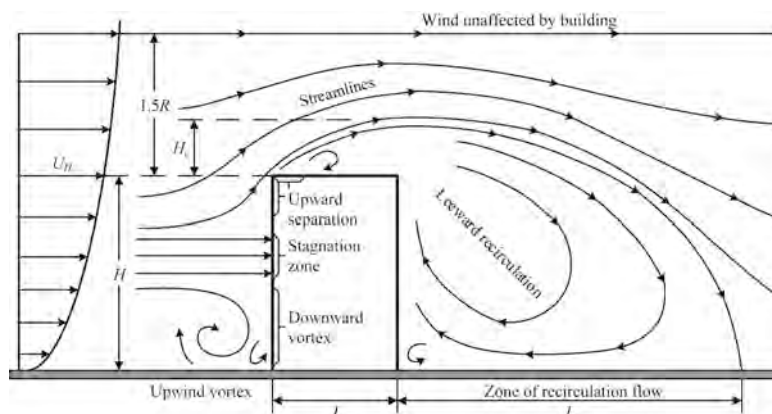


Figure F2.2 Wind movement about a tall building. *ASHRAE Handbook*, 2011, Section 45.3 as revised by MAO Jiachen and GAO Naiping in *Building and Environment*, Vol 94, Pt2, Dec 2015.

## 2.2 Seismic or Earthquake

Earthquakes produce dynamic lateral loads on buildings that vary in magnitude from mild tremors to catastrophic ground shaking. They are fundamentally different from wind force and are generated from ground movement at the base of a building. Hong Kong is fortunate to be in a region that is not prone to earthquakes. The subject of structural design in response to earthquake is therefore not part of the examination.

## 2.3 Lateral Ground Pressure and Hydrostatic Pressure

Horizontal pressure on a building's substructure results from the weight of the soil pressing laterally on the surfaces in contact with the soil. In addition there may be factors that can cause soil movement and an increased amount of pressure. If all or part of the substructure is below the water table, the water in the soil (water is heavy) presses laterally on the buildings sides. Both pressures increase with depth.

## 3.0 High-rise Building Structure

If we just consider height, currently (2024) there are 176 buildings in the world taller than 300m. When the Eiffel Tower was built in 1886-89, it became the first tower taller than 300m. The Council on Tall Buildings and the Urban Habitat defines a tall building as:

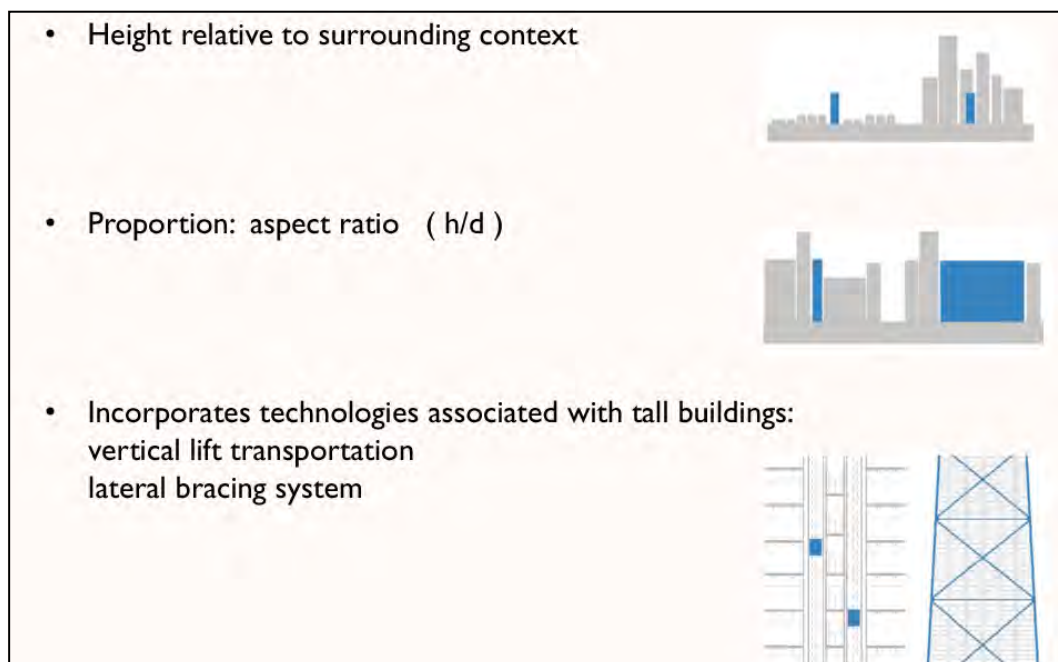


Figure F3.1 What is a tall building? *Council on Tall Buildings and the Urban Habitat.*

The aspect ratio defines the *slenderness* of a building. It is the height divided by the least dimension in width. So, for example, a rectangular building 40m x 20m in plan and 100m tall, has an aspect ratio of 5.

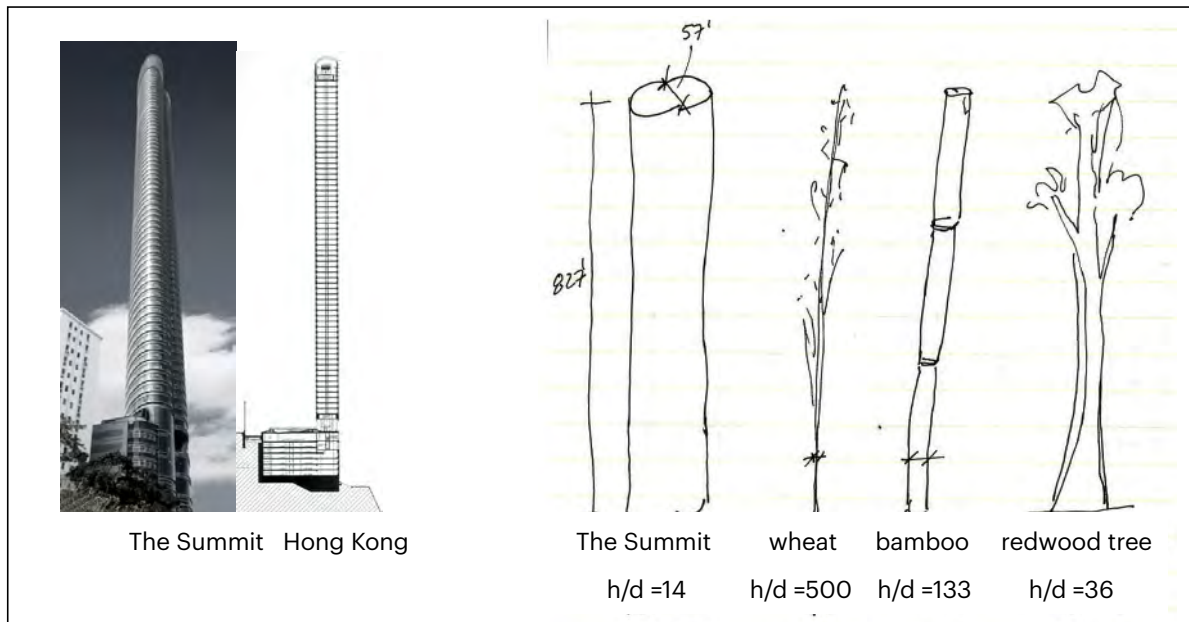


Figure F3.2 Aspect ratio

### 3.1 Structural response to lateral force

When a building reaches a certain height, the lateral forces of wind and earthquake play an increasing role in structural design considerations. For low and mid-rise steel framed structures the lateral stability provided by *frame-action* alone is usually efficient only up to about 10 stories. Above this threshold, other resisting systems such as diagonal bracing and shear walls come into play. R/C framing structures vary in their lateral force resistance: the flat plate and column framing system for example, is efficient for moderate vertical loads but the thinness of the floor plate is unable to handle the bending moments from large lateral forces that develop at the column-plate interface. Beam and slab systems are more effective in resisting lateral force induced bending.

As the height of a building increases, additional lateral stiffening systems must be incorporated. These include rigid cores (either r/c wall or braced frame), exterior diagonal bracing, outrigger trusses, tube configurations, etc. It has been demonstrated (see Figure 3.3 below) that for steel frame systems of a height of roughly 50 floors (200m), the weight of steel required for the lateral wind force stiffening is approximately the same as the weight of steel required for the vertical load steel (columns) or the weight of steel required for the floor framing (approximately 8 psf or 383 N/m<sup>2</sup> for typical spans of 8-10m).

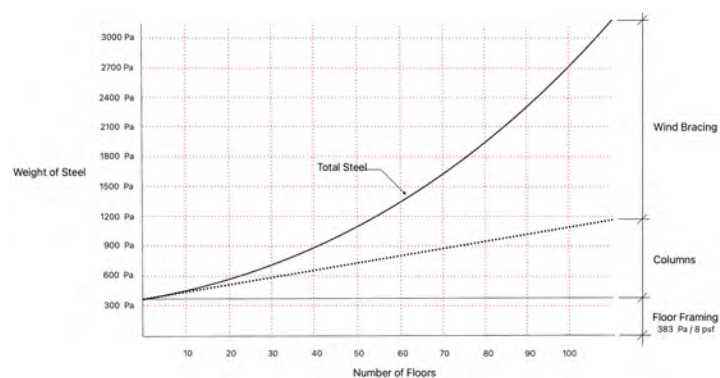


Figure F3.3 The weight of steel required in a multi-story steel frame building for different structural systems (wind bracing, columns, floor framing) versus the number of floors

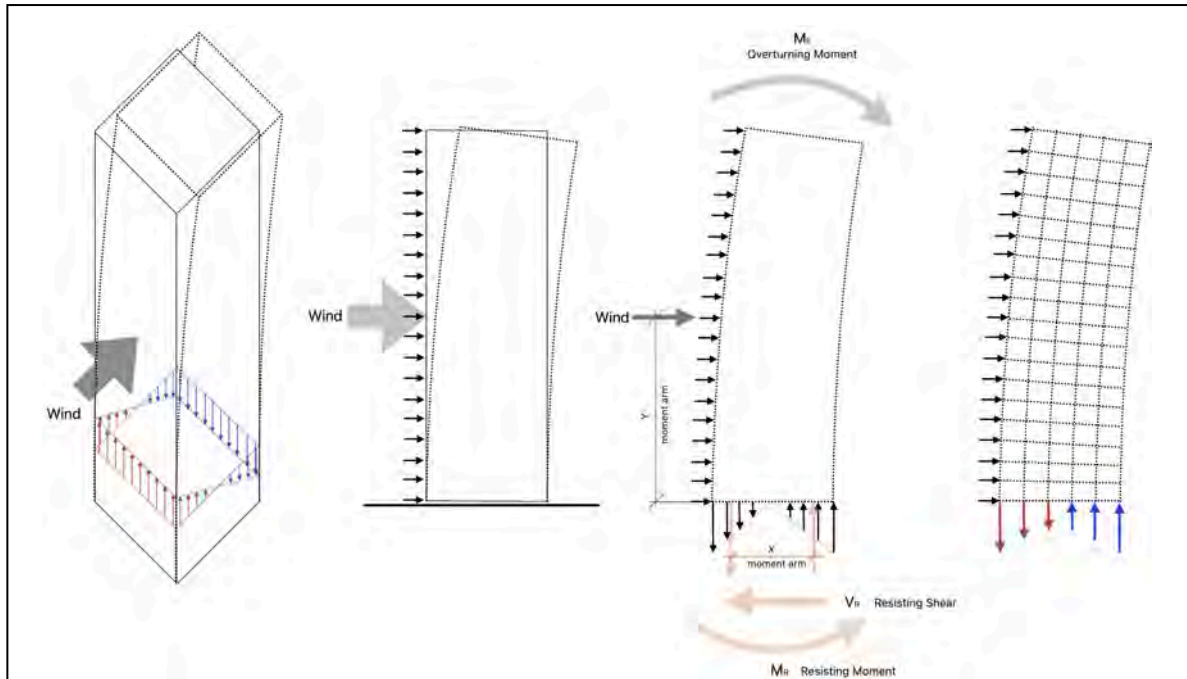


Figure F3.4 Cantilever behavior of tall slender towers.

Tall buildings that are relatively slender tend to behave as vertical cantilevers. That is, the *bending and deflection* caused by lateral wind loads is similar to a cantilever beam with vertical loading. The diagrams below try to explain the behavior. Wind force on the surface of the tower produces an over turning moment which must be resisted by an opposite moment generated at the base of the building. The edge of the building at the ground on the windward side will be in tension (as on the upper face of a cantilever beam) while the opposite side (leeward) will be in compression. Stresses at the center (the neutral plane) will be zero. The resultant force vectors of these stresses create a *force couple* (equal to the force times the distance between them (moment arm)). This is  $M_R$  or resisting moment.

In a high-rise building with frame structure, lateral wind force will produce either tension or compression in the columns depending on their position. Note that as the tower base becomes wider, the moment arm of the resisting moment,  $M_R$  becomes larger as well (hence generating a larger  $M_R$  with no increase in the column forces). This explains why a tower with wider base such as the Eiffel Tower, is more efficient in resisting wind load.

### 3.2 Stiffness / Rigidity

*Stiffness*, not strength, is the critical design parameter in very tall buildings. Lateral movement at the top of a tower due to wind forces must be controlled by the design of the tower's structural system. The movement back and forth at the uppermost portion of a tower due to wind is called *sway*. Tall buildings can be designed to sway as much as a meter (maximum deflection  $\leq h/500$ ) under extreme wind conditions. However, even small amounts of movement can be disconcerting to occupants. Such movements caused by sway can also damage non-structural elements such as piping, wall partitions, and window glass. Lift performance can also be compromised.

Structural design can minimize sway by increasing the buildings overall stiffness. Another method is the use of mechanical dampening systems. A *tuned mass dampener* is a large mass (e.g., concrete) near the top of the tower on a friction-less pad with springs



attached. The inertia of the mass resists movement initially and then absorbs energy in opposition to the return motion of the sway.

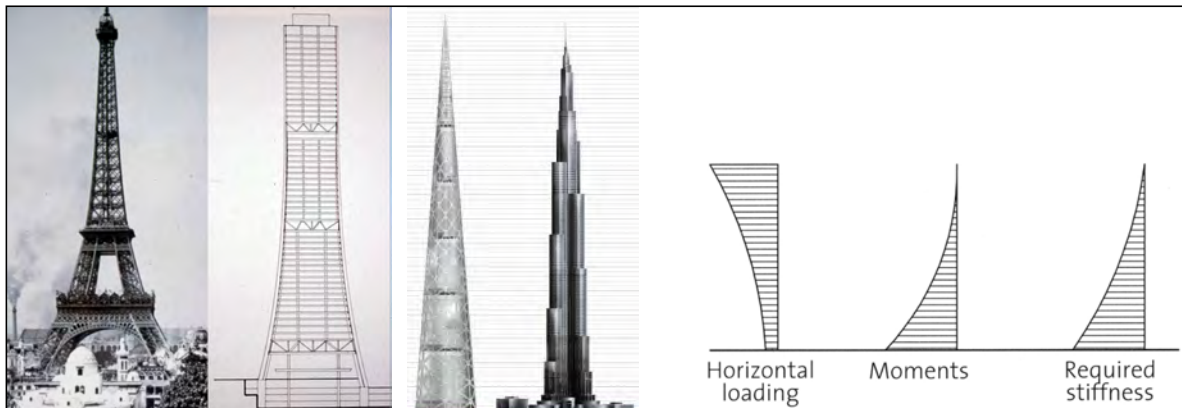


Figure F3.5 Shape of some tall buildings that directly reflect the overturning moment.

### 3.3 Types of Structural Systems

As buildings have evolved and become taller, engineers have developed new structural systems to resist the increase in loads, in particular, the lateral loads generated by wind. As previously noted, the amount of material by weight required by the vertical load supporting system (columns and walls) will equal the amount required by the lateral force resisting structure at a height of approximately 50 stories .

To accommodate the larger vertical gravity loads that accumulate as the height increases, columns have become both larger and stronger. Higher strength materials are used, especially in the columns at the base, and composite columns of steel and concrete have become standard. Ironically, the *number of columns* used has decreased as the columns in tall buildings become larger. For the lateral force resistance, new systems such as exterior truss bracing and tubular designs have emerged and are now standard solutions.

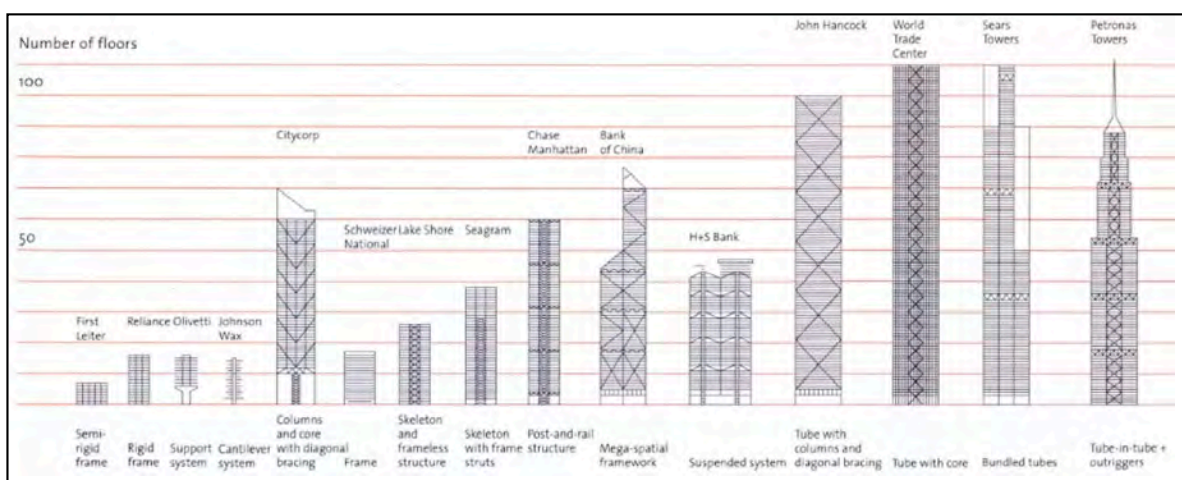


Figure F3.6 Comparative height of different structural systems for tall buildings. *High-rise manual: typology and design, construction, and technology*, ed. Eisele and Ellen Kroft, 2003.

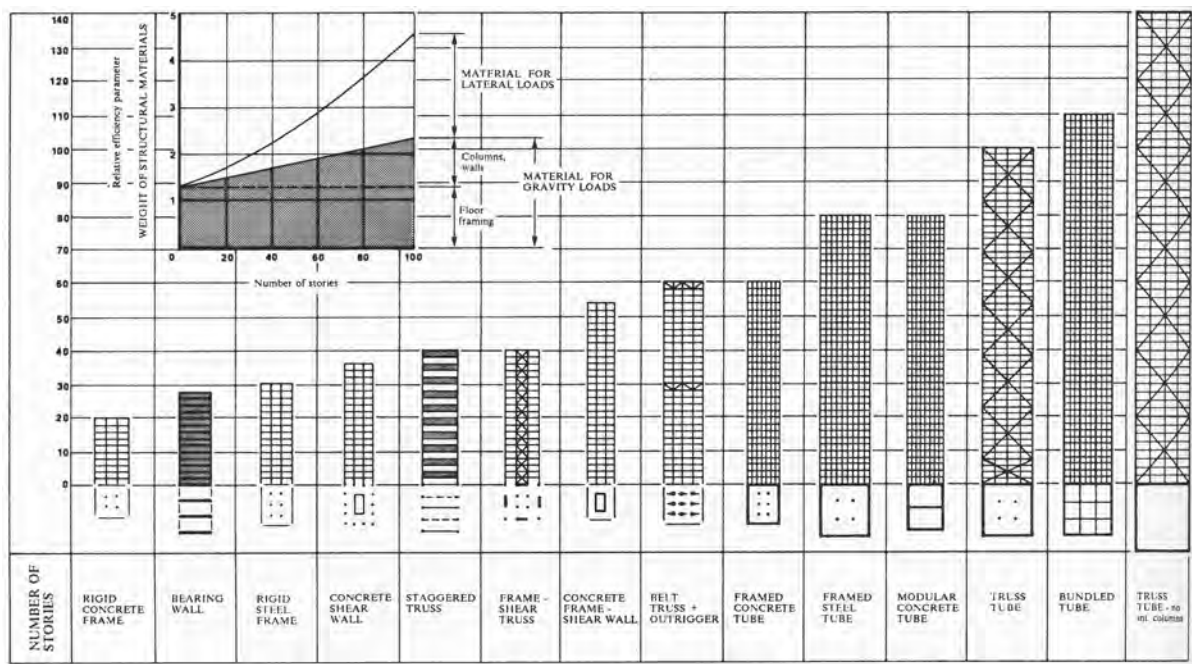


Figure F3.7 Types of structural systems for Hi-rise Buildings. *The Vertical Building Structure*, Schueller. Figure 7.1 p446

### 3.3.1 Bearing Wall

Load bearing walls of masonry, predominantly brick, constituted multi-story construction prior to the late 19c. These structures attained their maximum height with the Monadnock Building in Chicago in 1891 (66m tall). Besides being the last of the high-rise masonry buildings (its wall structure at the base measures 1.8m in thickness) it was one of the first tall buildings to incorporate a *portal system* of bracing to resist lateral forces. The steel frame and later the reinforced concrete frame practically replaced wall structure for all tall buildings for almost 100 years. Today, with composite steel and concrete design, wall structure is returning as it has the advantage of working as both *load bearing* wall and *shear* wall.

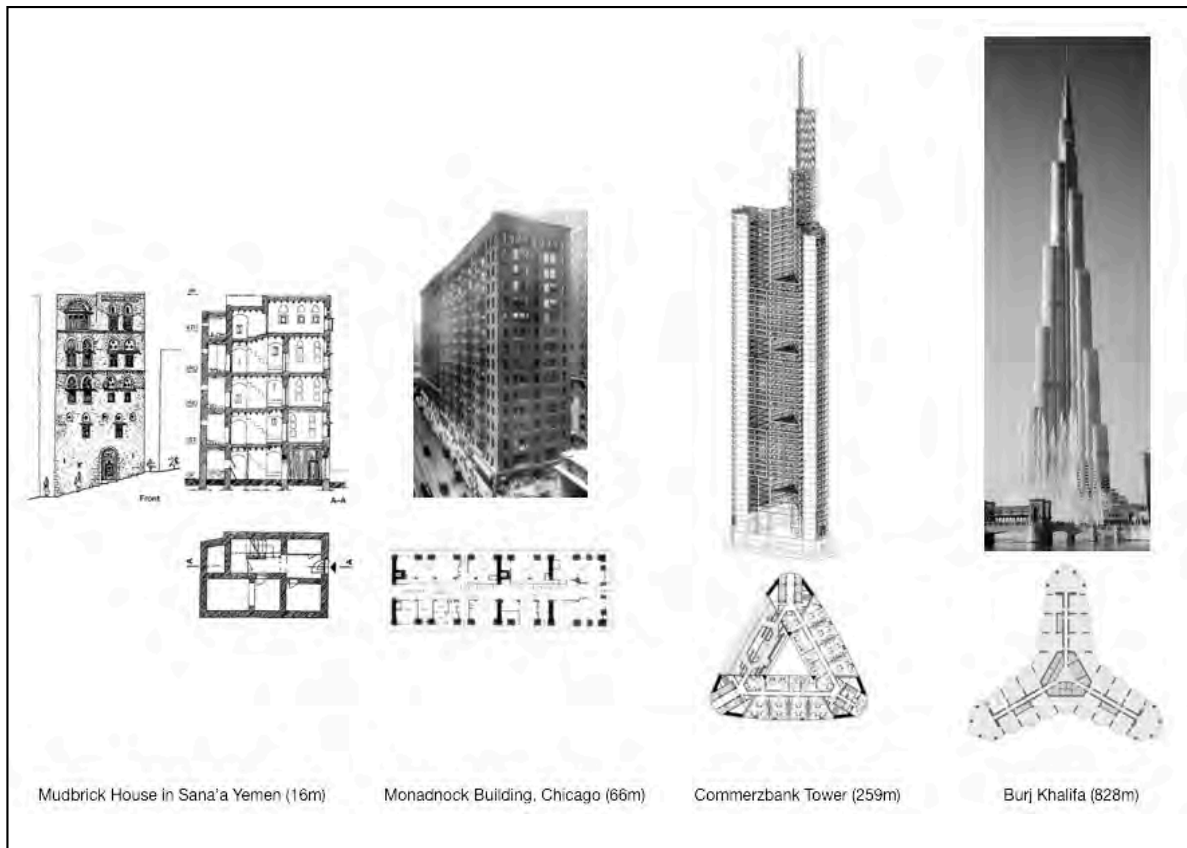


Figure F3.8 Evolution of Bearing Wall Structure in Tall Buildings

### 3.3.2 Bearing Wall and Core

The Hong Kong Club (Harry Seidler 1985) is an example of a bearing wall structure with a shear wall core bracing the building in the rear. Four curved concrete walls at the corners support the shaped, long-span pre-stressed beams that make the facade of the building. The Knights of Columbus Building (Roche Dinkaloo 1969) is another example. Here the bearing walls are also lift cores; cylindrical shafts located at the four corners with a rectangular core in the centre. Floor framing uses long-span steel girders between the central core and the corners. At 252m, the High Cliff Tower (DLN 2003) in Hong Kong was possibly the tallest shear wall building constructed. Its narrow width (aspect ratio of 1:20) is traversed by five load bearing shear walls that provide the lateral resistance. The tower also has a dampening device that uses the water tank of the emergency fire suppression system (located at the top level of the tower) to minimize the amount of sway.

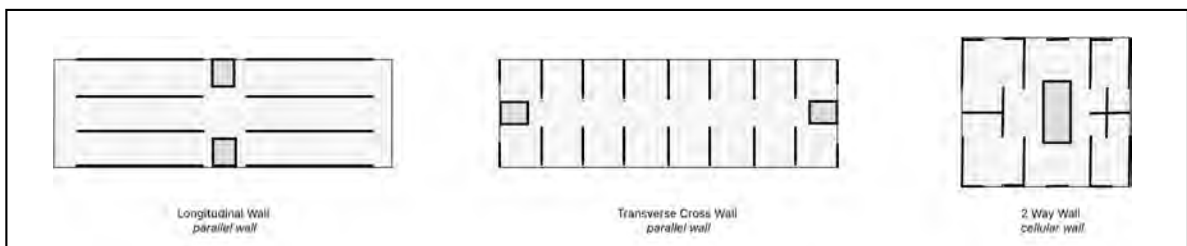


Figure F3.9 Bearing wall and core structures. Types of wall.

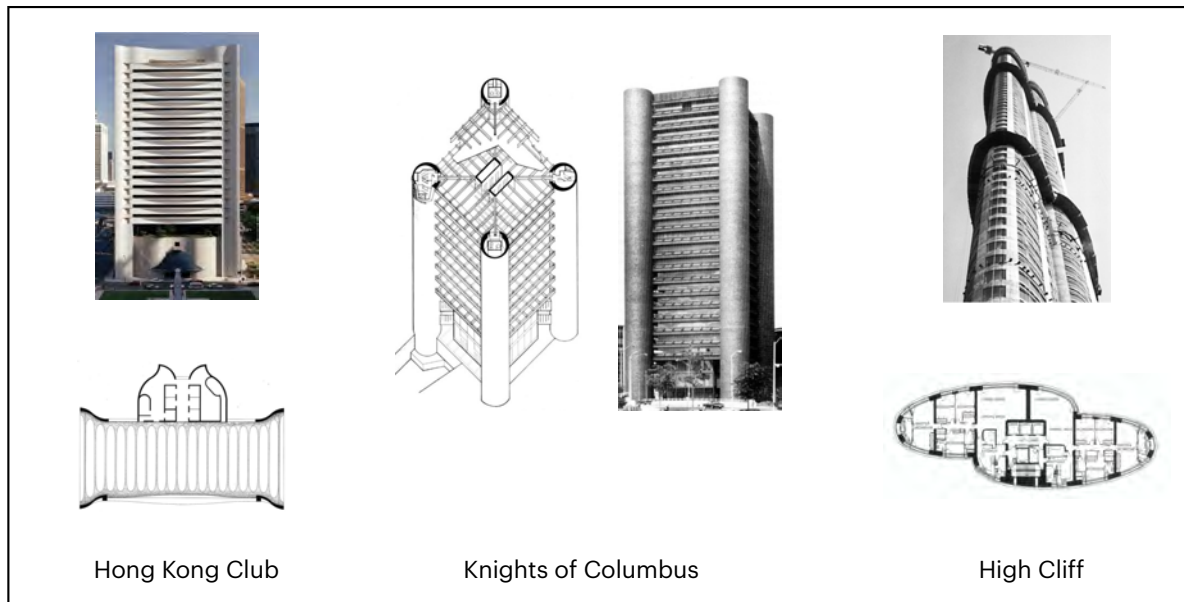


Figure F3.10 Bearing wall and core structures. Examples.

### 3.3.3 Cantilever Tower

A type of bearing wall structure is a *cantilever tower*. Basically a core with floors that are cantilevered or supported by suspension, the concept was introduced by Frank Lloyd Wright in the Johnson Wax Headquarters Research Tower (1950) and later the Price Tower in 1956. The idea of a strong central core (analogous to a tree) supporting both the vertical gravity loads and acting as a shear core against lateral loads was also adopted for the Standard Charter Bank of Johannesburg (1968). The concept is attractive; a core can be rapidly constructed and then floors built from the top down. The flaw in the system however, is that all the vertical structure is concentrated at the center of the plan; the least efficient location for bending resistance. This can be demonstrated by comparing the bending resistance of various configurations of column mass as measured by the moment of inertia of bending about the x-axis (or y-axis).

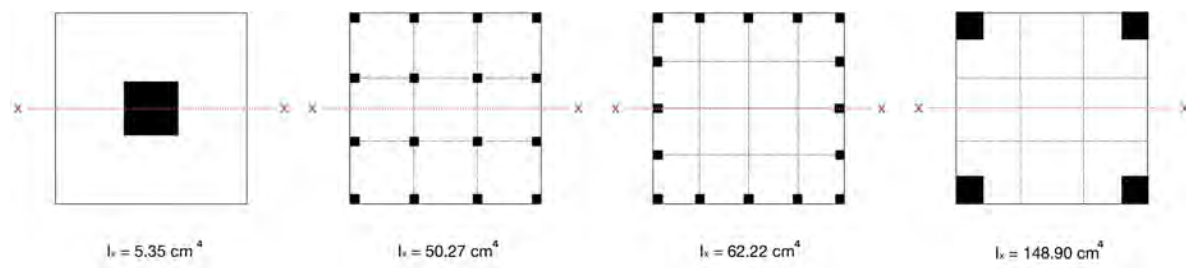


Figure F3.11 Comparison of configuration of the bending resistance mass (equal areas of 8cm<sup>2</sup>)



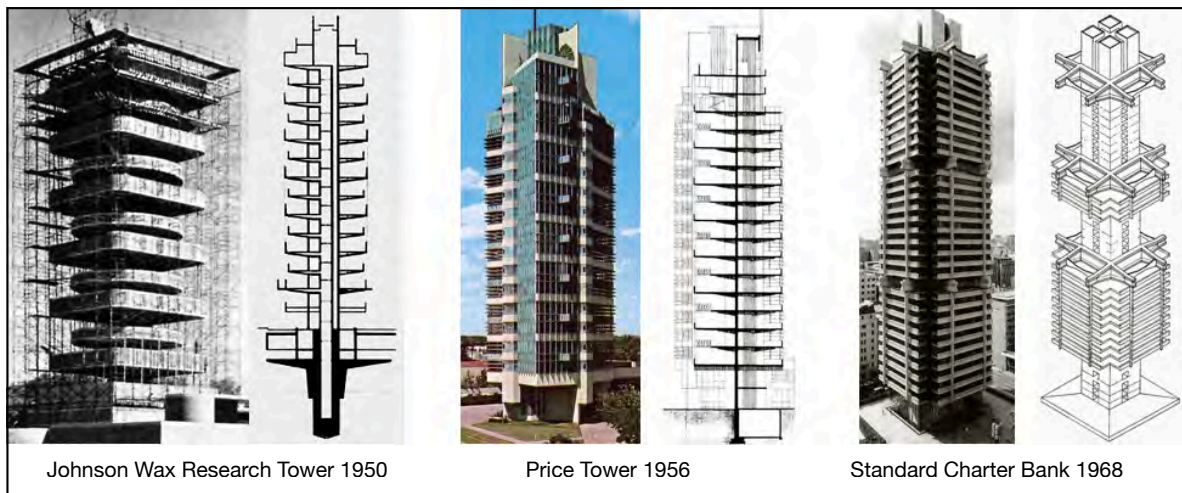


Figure F3.12 Cantilever Tower. Examples.

Structural systems in which floor loads are *transferred* to either main structural columns on the perimeter or to the core (as in cantilevered towers) have been used in towers such as the Kahn's University of Pennsylvania Medical Research Towers and Foster's Hong Kong Bank (HSBC). Theoretical possibilities were summarized by Jack Zunz, engineer.

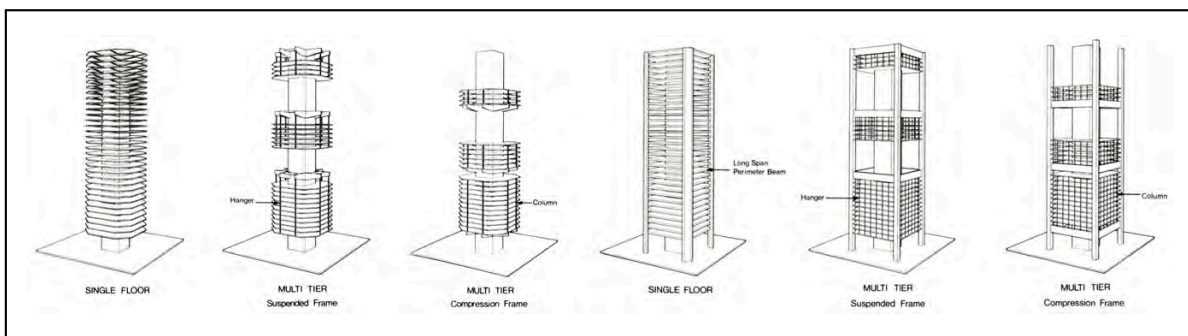


Figure F3.13 Theoretical possibilities of transfer structures in tall buildings. Jack Zunz.

### 3.3.4 Frames

Depending on the type of beam-column connections employed, frames are categorized as either pinned (hinged), semi-rigid or rigid. Pin connected frames are those in which the beams are simply connected to the columns by 'shear connectors' such as web plates and angles. These offer no moment resistance and beams can rotate at the joint, causing the frame to offer no bending resistance to lateral loads. If columns are continuous and fixed at their base, there is some resistance due to the cantilever stiffness of the columns. But in general, frames with simple connections (hinge type) must rely on a shear resisting structure such as a core or diagonal bracing for lateral resistance and stability.

Rigid frames are those with fully rigid beam-column connections in which the beams and columns remain at 90° perpendicular to each other even as the joint itself rotates slightly. This forms a very stiff frame capable of resisting lateral force by means of the bending stiffness of the beams and columns. Rigid frames exhibit a deformation (or distortion) in two ways: *cantilever bending* wherein all the columns bend a certain amount away from the direction of the load, and *racking*, in which the shear distortion of the frame results in a sideways deflection. The combination of these two actions produces the total lateral deflection of the tower, or *drift*, measured at the top.

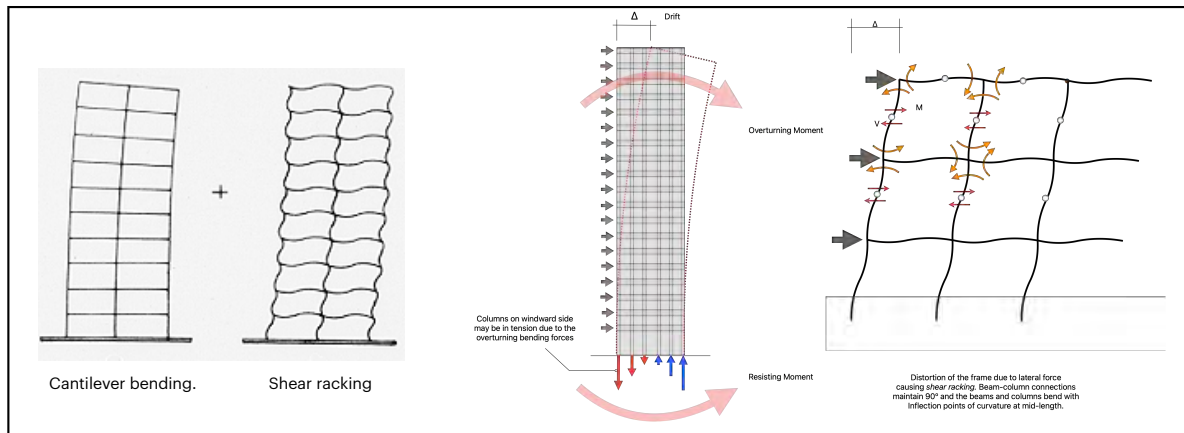


Figure F3.14 Rigid frame deformation as a combination of cantilever bending and shear racking.

Semi-rigid frames are those in which the connection of the beam to column is by a combination of a web angle and top and bottom flange angles (normally bolt connections although the flange connection angles may also be welded. See Figure 1.1 Steel framing Connection Types). Although there is some rotation of the beam at the joint, semi-rigid frames can resist lateral forces up to a certain degree. In general frames with rigid or semi-rigid connections are limited to about 30 stories.

Reinforced concrete frames have rigid beam-column connections by virtue of their monolithic construction. Reinforced concrete frames with flat plate floor construction have some stiffness, but not as much as a beam slab or waffle slab floor construction. They are economical for approximately 15-20 floors depending on the floor to floor height.

The Lake Shore Apartments (Mies van der Rohe 1947) was one of the first residential steel frame towers built. It has no shear core and relies on rigid frame stiffness for lateral stability. The Stanhope Building in Hong Kong is a typical, concrete frame structure with bays that are approximately 6-7m square and subdivided into three secondary bays. The monolithic concrete frame and slab construction provide more than adequate stiffness to resist lateral forces. The depth of the building also contributes to its overall stiffness.

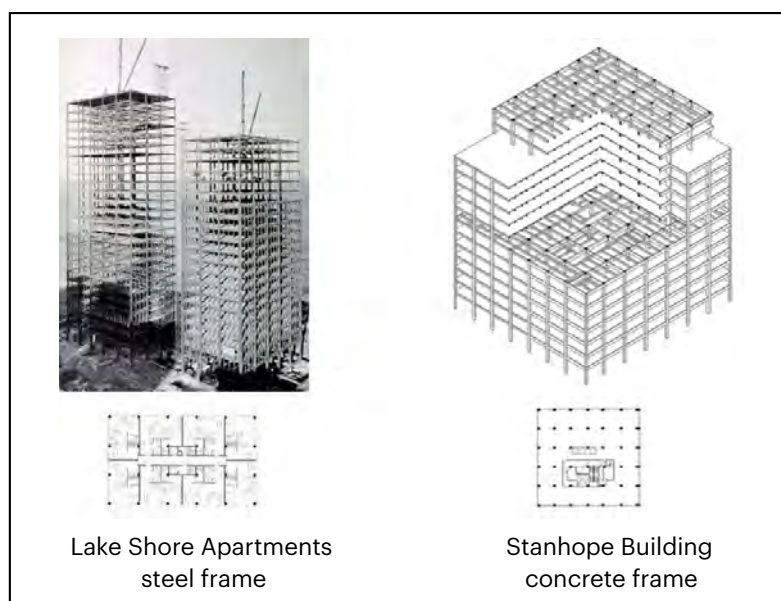


Figure F3.15 Rigid frames with no shear walls. Examples.

### 3.3.5 Frame plus Shear Wall

With the addition of a rigid core, a frame structure has a shear wall in the structure of the core that can resist lateral loads. Shear walls that are not part of the core may also be used. The wind forces are transmitted to the core and shear walls either by the beams or through a diaphragm (rigid) floor.

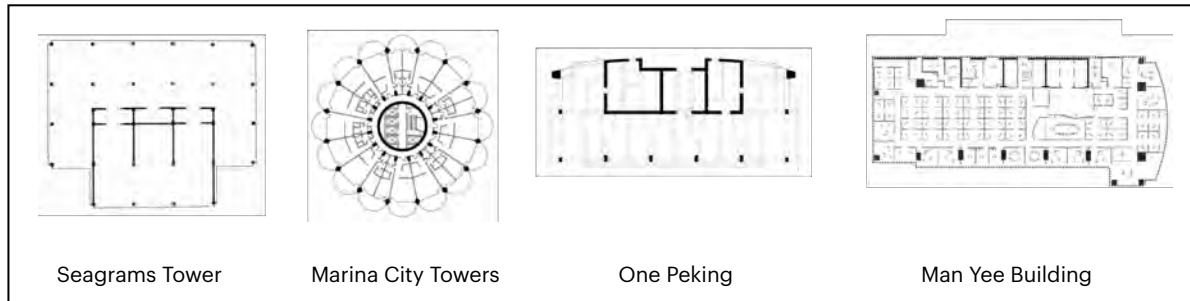


Figure F3.16 Rigid frames with shear walls. Examples.

### 3.3.6 Inter-spatial Truss and Staggered Truss Systems

An innovative method of framing multistory buildings was developed in the mid-60's and involves the use of long-span, floor deep trusses *staggered* in one of two ways:

- i) Trusses span between columns but skip every other floor. The span of the secondary framing determines the spacing of the columns. Trusses support the floors on both top and bottom chords. Advantage: every other floor is open and column free.
- ii) Trusses are staggered in their position on alternate floors. Column spacing is half the spacing between trusses. Trusses support floors on both upper and lower chords. Advantage: wider spacing between trusses (but circulation must go through the trusses).

Trusses spanning the full width of the shorter dimension of the building stiffen the frame in that direction. In the longer direction another lateral force resisting mechanism is required, either rigid framing or some form of shear wall or shear bracing (e.g., diagonal bracing).

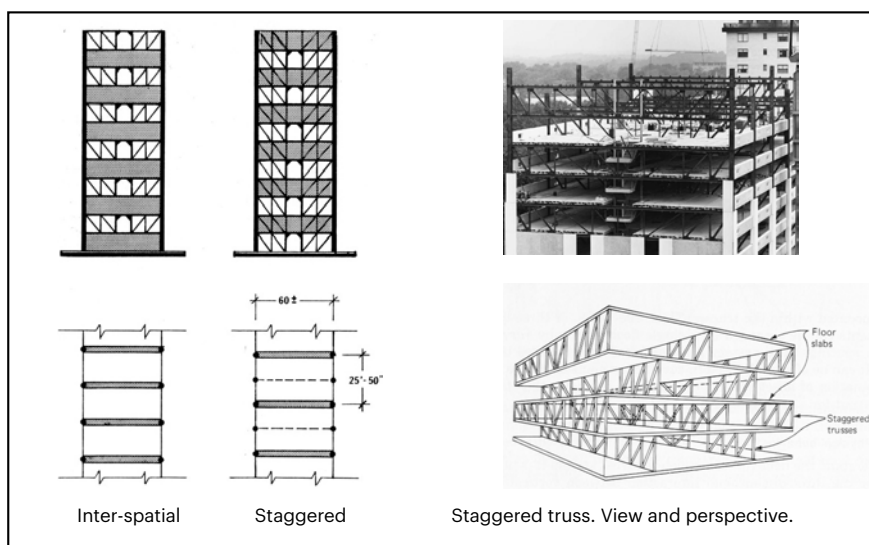


Figure F3.17 Inter-spatial and Staggered Truss Systems. *High-Rise Building Structures*, Schueller.  
Figure 5.13 p93

### 3.3.7 Frame plus Shear Truss

A steel frame can gain additional stiffness with the introduction of *vertical shear truss bracing*. The bracing may be hidden inside a non-structural core or placed on the exterior of the building and expressed as part of the envelope aesthetic. The diagonal bracing uses the columns of the frame as chord members to create a truss. Above 40 stories a trussed or diagonally braced frame is economical. These trussed elements are usually symmetrically located and are analogous to shear walls. Depending on the amount of trussing, high-rise buildings can achieve significant height with this system.

The Hearst Building and the Swiss RE Building represent the most current development in the use of diagonal bracing. These two towers eliminate the vertical column structure altogether and substitute a peripheral *diagrid* structure that carries both the vertical and the lateral loads.

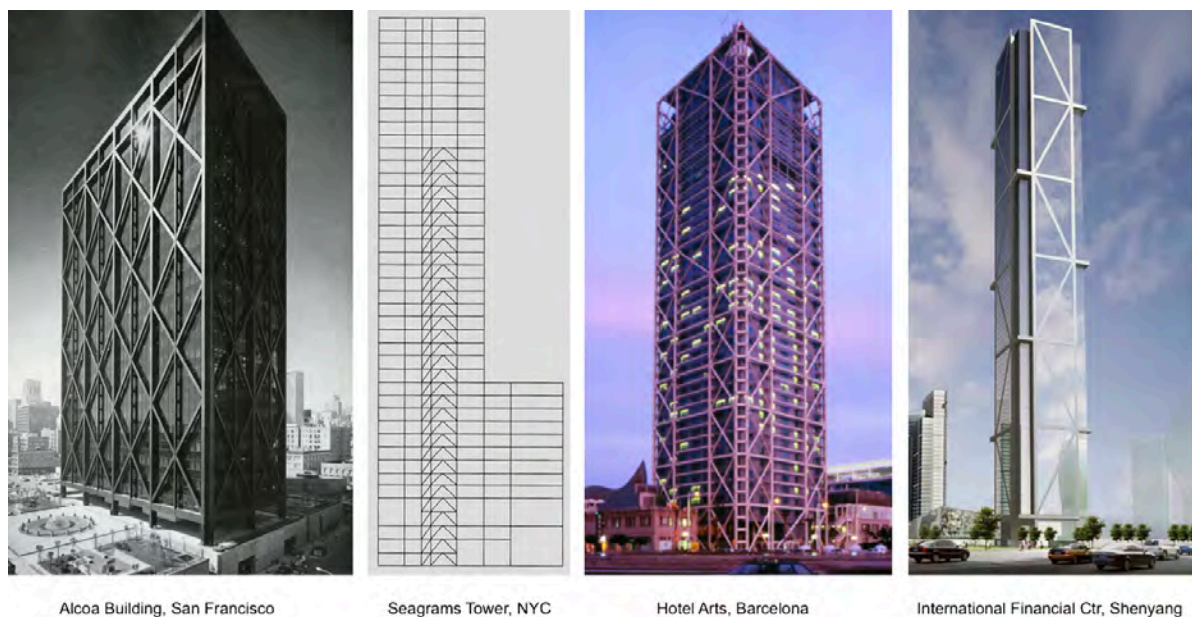


Figure F3.18 Shear Truss Frames. Examples.

### 3.3.8 Outrigger and Belt Truss

An outrigger and belt truss system uses deep stiff trusses at the mid or upper level of a tower to connect the perimeter vertical columns of the frame to a rigid central core. By effecting this connection, the normal bending configuration of a frame-core system (cantilever bending plus rigid frame side-sway) is altered into a more complex deformation mode that is stiffer, resulting in less overall bending moment and subsequent less lateral displacement.



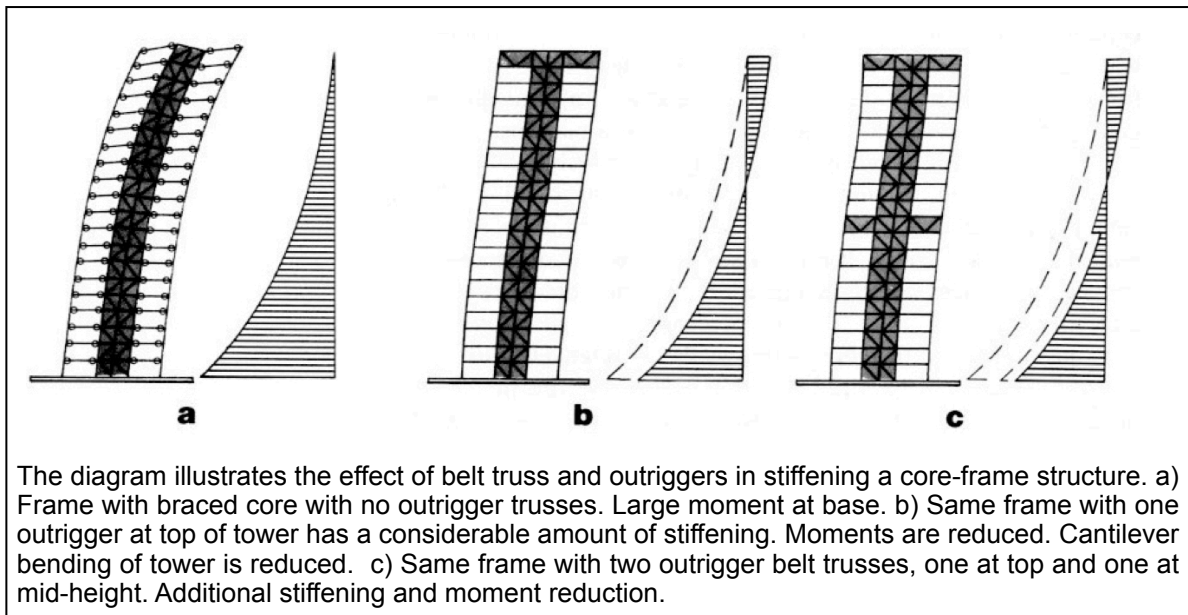


Figure F3.19 Comparison of Outrigger and Belt Truss Structures with Braced Core only.

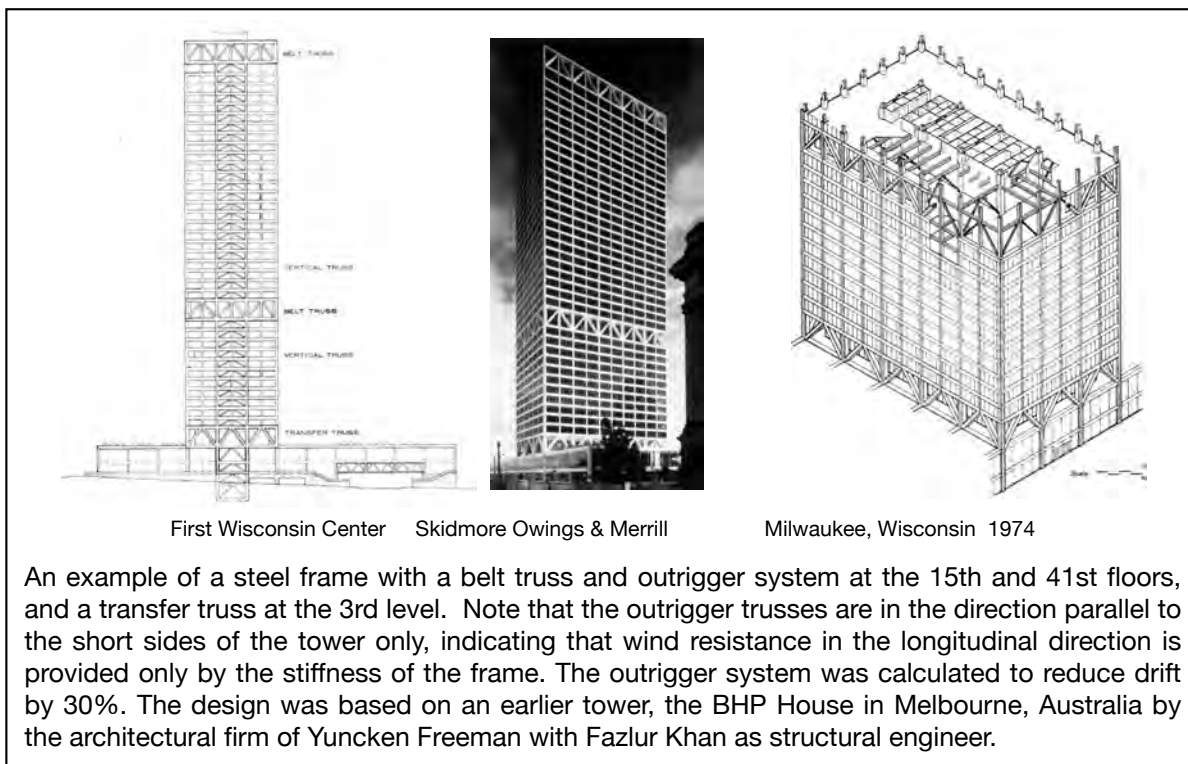


Figure F3.20 Outrigger and Belt Truss Structure. Example.

### 3.3.9 Tube Structural Systems

In the 1960's engineers at SOM Architecture led by the structural engineering innovator, Fazlur Khan, developed a new approach to high-rise structure that came to be known as

tube structure. It was observed that a cantilever structure (a tower is essentially a vertical cantilever beam) was most efficient if it had most of the material of the section in the outer portion away from the center of gravity. For a beam this means the extreme top and bottom surfaces since a beam bends in only one direction. But a tower has to respond to lateral wind force from all directions. Therefore a *square tube* was chosen as the best configuration for the section. Apparently Khan was inspired by the structure of bamboo.

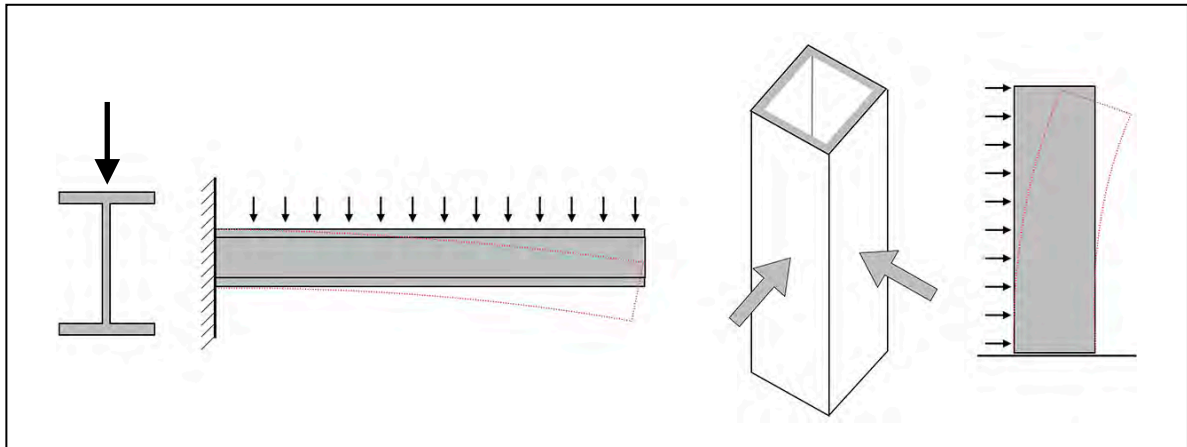


Figure F3.21 Analogy of a tall building bending and a cantilever beam

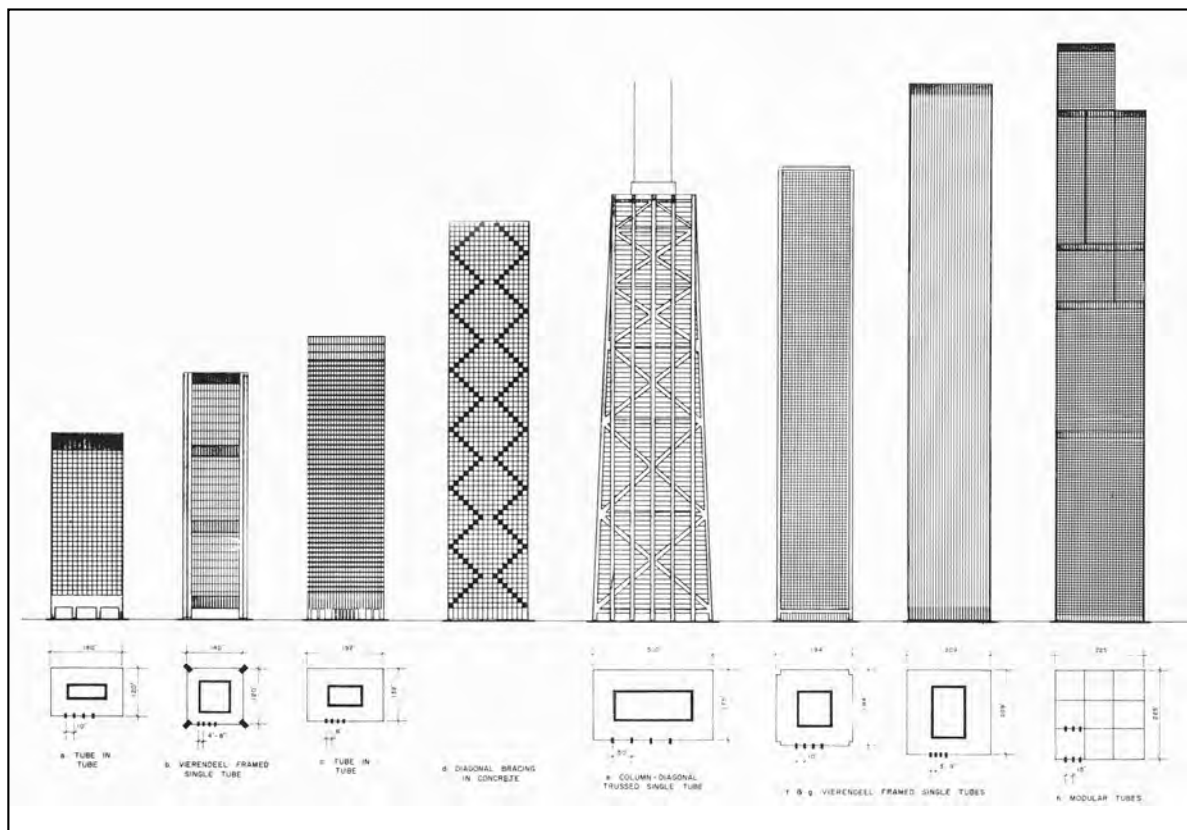


Figure F3.22 Variation of Tube Structure. *High-Rise Building Structures*, Schueller. Figure 5.20 p102



Figure F3.23 Tube Structures. Examples.

- Framed tubes:** close spacing of columns on the exterior surfaces forming a closed plan shape, equivalent to a vertical silo or tube with fenestration between columns and beams. More columns on the perimeter places more material away from the center to resist the bending force caused by lateral wind loads more efficiently. The fenestration becomes more solid and dense with less window area. The earliest examples of this approach were designed by SOM: the Chestnut-Dewitt apartment building (Chicago, 1965) and the Brunswick Building (Chicago, 1966). The exterior column spacing of the Chestnut-Dewitt Apartment Building is just 1.7m (5' 6") c.c. while that of the Brunswick Building is 2.87m (9' 4") c.c. The ultimate framed tube structure was the World Trade Center (Yamasaki/Leslie Robertson, NYC 1974). The window openings in the Trade Center towers were only 19in (48cm). The twin towers reached a height of 420m. Framed tubes are also designed in concrete. The One Shell Plaza tower in Houston, Texas (SOM, 1971) was an early reinforced concrete tube structure at 220m in height.
- Truss tube:** similar to the framed tube but the column spacing is wider and the columns are connected to a system of diagonals in the exterior surface. The classic trussed tube is the John Hancock Center (SOM, Chicago 1970). The diagonals here, in addition to acting as bracing members, also carry a portion of the gravity load thereby constantly being in *compression* (except at the very top, any tension that develops in the diagonals due to acting as cross bracing to resist lateral force, will be neutralized by the compression from the vertical loading). The John Hancock Tower was one of the most efficient tall building structures ever made. At 100 stories, the weight of steel was less than 30psf (1.44kN/m<sup>2</sup>). Compared to other hi-rise structures of the time (Chase Manhattan Tower, NYC at 60 stories/55psf and US Steel Tower, Pittsburg at 40 stories/30 psf), Hancock was a true outlier.

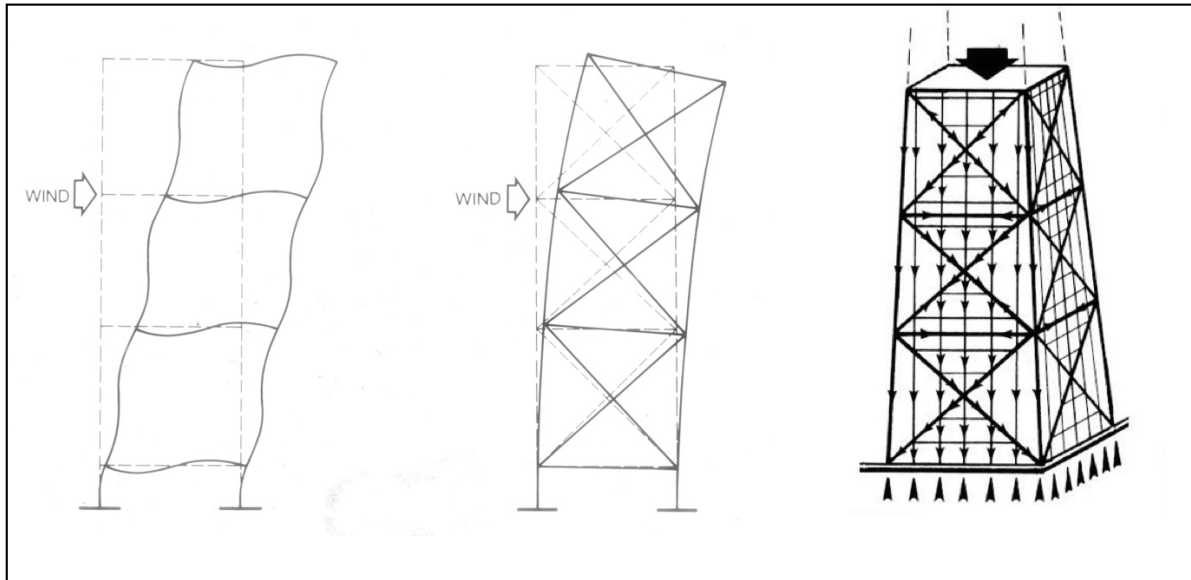


Figure F3.24 Behavior of a trussed tube structure

- **Bundled or modular tubes:** framed or trussed tubes grouped together like cells to compose the overall building shape. The tube walls between cells are common to adjacent cells. The most famous bundled tube tower is the Willis Tower (formerly the Sears Tower, SOM, Chicago 1974, 442m). With a footprint of 70m x 70m the Willis is one of the deepest towers built. The concept of the bundled tubes allows the tubes to discontinue at different heights, giving the tower its unique asymmetrical profile.
- **Space-frame braced tubes:** The Bank of China in Hong Kong is a five section super scaled vertical space frame. The truss modules are 13 stories in height and floor loads are transferred to deep truss beams at the bottom of each module. The frame is designed with a central column above the 25<sup>th</sup> floor that allows portions of the module to drop-off in the manner of the Willis Tower. This gives the Bank of China its distinctive profile. At 372m the BOC was the tallest building in Asia at its inauguration in 1985.

### 3.3.10 Composite Structural Systems

Composite systems generally refer to mixed reinforced concrete and structural steel structures with concrete shear walls or concrete framed tubes combined with various structural steel framings. Introduced by the structural engineer William LeMessurier in the design for the Bank of the Southwest (1982), the configuration of eight composite mega-columns on the outside perimeter tied back to a concrete shear core by outriggers (steel trusses) has since become a common structural system for tall buildings in the 400m range. The IFC2 in Hong Kong (Rocco Design Architects Associates Ltd. + Pelli Clark Pelli, 2003) is a composite mega-column structure. The four main structural features of the system include:

- A reinforced concrete core. Transmits vertical loads and resists lateral forces.
- Eight composite columns. The columns are arranged on the perimeter of the building and align with the outside planes of the core. No columns are at the corners, a feature which provides high-premium column-free corner offices.
- Outrigger trusses connecting to the mega-columns and a belt truss. The strength and stiffness of these trusses causes the core to be active in the resistance to bending force. The concept is similar to a “tied mast”.
- The distance between the pairs of columns on each side (24m) requires a deep girder beam; a key component of the external frame.



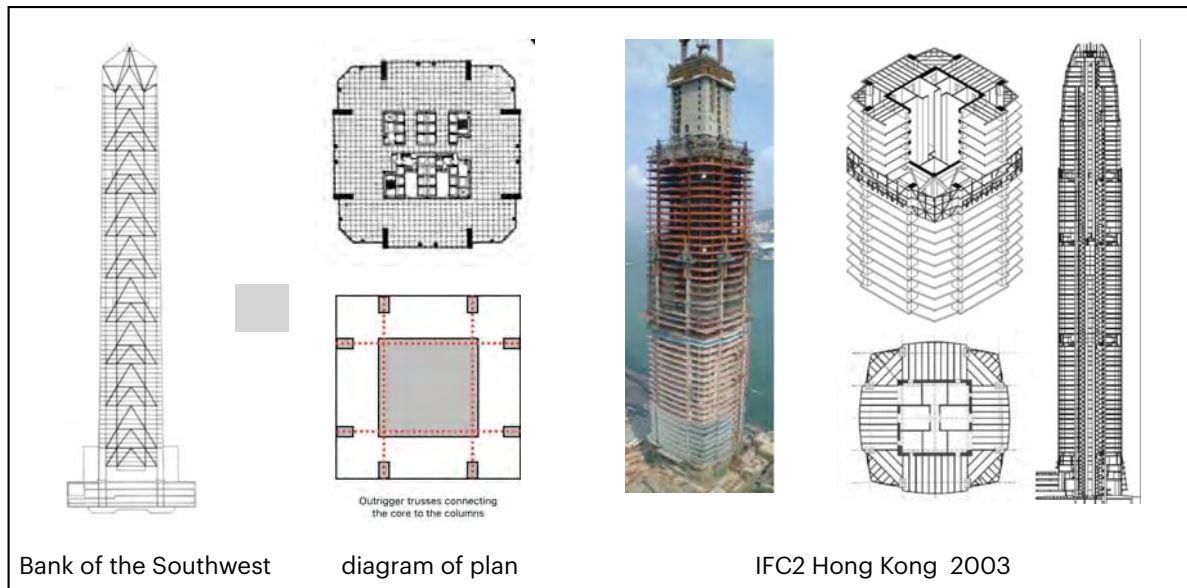


Figure F3.25 Composite Mega-column Structure.

### 3.3.11 Megaframe Structure

A megaframe is a *vierendeel* frame where the in-plane depths of horizontal and vertical members are large and contain plane trusses several stories deep or several bays wide. There have been several proposals for a 100 story plus megastructure. Oddly perhaps the HSBC Tower in Hong Kong (Foster 1985) at just 43 stories represents the characteristics of a megaframe best. Its eight vierendeel four-shaft cluster columns with trusses supporting groups of floors in suspension with column free spaces below each grouping seems to meet the definition of a megaframe. We might also consider Myron Goldsmith's 1953 MS Thesis project at IIT as a visionary project identifying the features of the megaframe structure. Here a profiled concrete megaframe supports floors at each level, with an equal number of floors suspended as well as compressed, that is, built as a compression frame. Steel framing is proposed for these secondary interior frames.

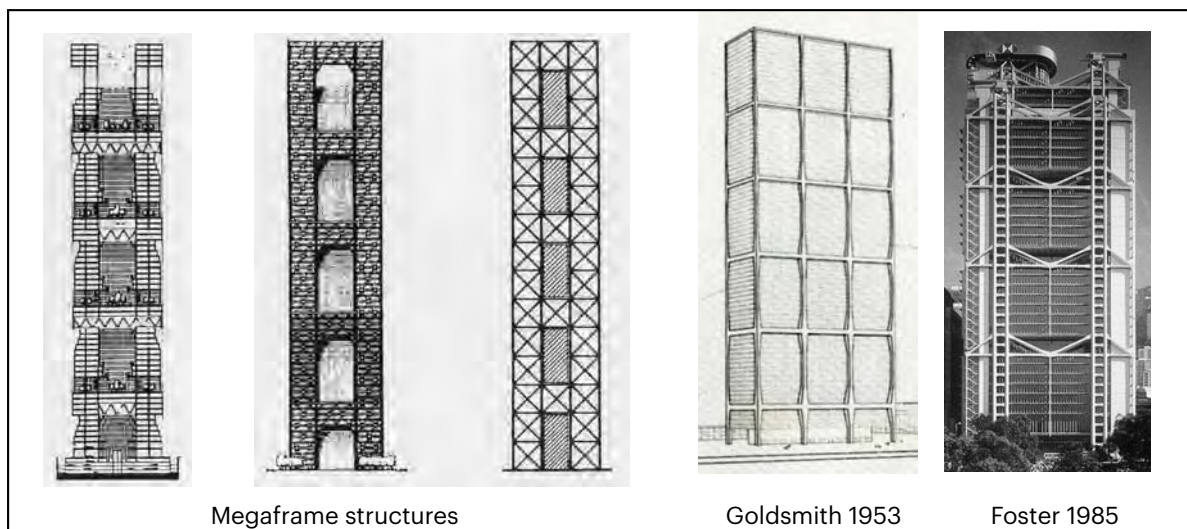


Figure F3.26 Megaframe Structures. Examples.

## REVIEW QUESTIONS

- Is the net wind pressure on a sunshade greater or lesser if it is held away from the envelope with an airspace between?
- What are some parameters that determine the force of the wind on a building?
- As buildings get taller, the amount of resistance to wind that a building must provide also increases. At approximately how many stories is the weight of steel that carries the gravity loading (columns) equal to the amount of steel required for lateral resistance?
- Which system provides more lateral stiffness? An interspatial truss system or a staggered truss system?
- What is the limiting condition in a cantilever tower?
- What is the most common structural system for an 80 story+ tower today? Why is it the favorite of developers of office towers?
- How does an outrigger and belt truss system stiffen a tall building?
- What are some disadvantages of tube structures? (Hint: view, ground level access, floor framing)
- Explain the concept of a tube structure.
- The John Hancock Tower uses only 29 psf total weight of steel? How does it do this?
- What makes a shear frame work? What key feature/s allows a shear frame to resist wind loads?
- Describe the different types of transfer structures proposed by Jack Zunz. Can you think of actual buildings that use each one of the different schemes?

## SELECTED REFERENCE

- 1) *Structures*, Daniel L. Schodek and Martin Bechthold, 2014 7<sup>th</sup> Ed., Pearson.
- 2) *The Vertical Building Structure*, Wolfgang Schueller, 1990. Van Nostrand Reinhold.
- 3) *High-Rise Building Structures*, Wolfgang Schueller, 1977, John Wiley & Sons.
- 4) *Building Structures Illustrated*, Francis D.K. Ching, Barry Onouye and Douglas Zuberbuhler, 2014 2<sup>nd</sup> Ed, John Wiley & Sons.
- 5) *Code of Practice on Wind Effects in Hong Kong*, 2019. Hong Kong Buildings Department.
- 6) *High-rise manual: typology and design, construction, and technology*, ed. Johann Eisele and Ellen Kroft, 2003, Birkhauser.